# Trigonometric & Polar Functions

AP Precalculus ▪ Unit 3

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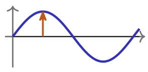




## What we will cover



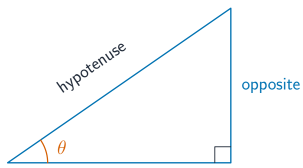
# 1 Periodic phenomena



## Periodic phenomena

A **periodic** 周期 function repeats its output values over equal input intervals – tides, daylight hours, a rotating wheel.

> Period and **frequency** 频率 are reciprocals. $\sin\theta$ has period $2\pi$.






## Sine, cosine and tangent on the unit circle

For a point $(x, y)$ at distance $r$ along the terminal ray:

$$\cos\theta = \frac{x}{r}, \quad \sin\theta = \frac{y}{r}, \quad \tan\theta = \frac{y}{x}.$$

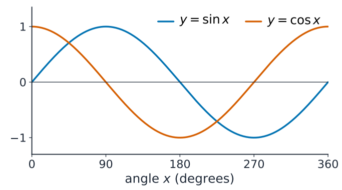
> $\tan\theta$ is the slope of the terminal ray – which is why it is undefined at $\theta = \frac{\pi}{2}$.

*Ferris color*

# 2 Sinusoids

## Amplitude, midline and period

- **Amplitude** 振幅 $=$ half the distance between max and min
- **Midline** 中线 $y = d =$ the average of max and min
- Period $= \dfrac{2\pi}{b}$






## Worked example – read a sinusoid

$f(\theta) = 3\sin(2\theta) + 1$

Amplitude $3$; period $\dfrac{2\pi}{2} = \pi$; midline $y = 1$. So the maximum is $1 + 3 = 4$ and the minimum is $1 - 3 = -2$.

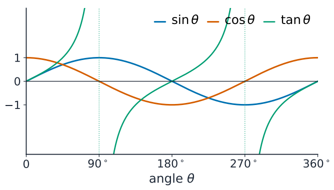
The general form is

$$a\sin\left(b(\theta + c)\right) + d.$$

> $a$ sets amplitude, $b$ the period, $-c$ the phase shift (a $+c$ shifts left), $d$ the midline.



## Transformations and data modelling




To model periodic data (tides, daylight, temperature): read the **period** from where the pattern repeats, get the **amplitude** and **midline** from the max and min, and set the phase shift so a peak...

> Technology fits a sinusoidal regression.

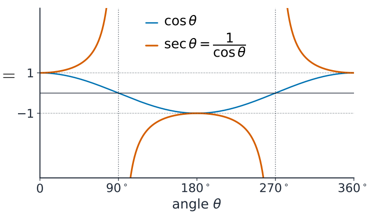
# 3 Inverse trig & equations



## Tangent, and the reciprocal functions

$$\sec\theta = \frac{1}{\cos\theta}, \quad \csc\theta = \frac{1}{\sin\theta}, \quad \cot\theta =$$

> $\tan\theta$ has period $\pi$ (not $2\pi$) and a vertical asymptote wherever $\cos\theta = 0$.






## Inverse trigonometric functions

Sine, cosine and tangent are not one-to-one, so each inverse needs a restricted domain.

> An inverse trig function returns an angle – and only the one in its restricted range.

- $\arcsin$: range $[-\frac{\pi}{2}, \frac{\pi}{2}]$
- $\arccos$: range $[0, \pi]$
- $\arctan$: range $(-\frac{\pi}{2}, \frac{\pi}{2})$

## Worked example – a trigonometric equation

**Solve $2\sin\theta = 1$ on $[0, 2\pi)$**

$\sin\theta = \frac{1}{2}$. On the unit circle sine is positive in the first and second quadrants:

$$\theta = \frac{\pi}{6} \quad \text{or} \quad \theta = \frac{5\pi}{6}.$$

Over all reals, add $2\pi k$ to each.

> A trig equation has infinitely many solutions – the calculator gives you one. Add the period, and find the second quadrant solution yourself.



## Equivalent representations – the identities

**Pythagorean**: $\sin^2\theta + \cos^2\theta = 1$

**Sum**: $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ $\quad \cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

**Double angle**: $\sin 2\theta = 2\sin\theta\cos\theta$

> An **identity** 恒等式 is true for every $\theta$ – use one to rewrite an expression into a form you can solve.

# 4 Polar

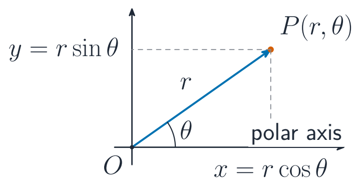


## Polar coordinates

**Polar coordinates** 极坐标 give a point by distance $r$ and angle $\theta$:

$$x = r\cos\theta, \quad y = r\sin\theta,$$

$$r = \sqrt{x^2+y^2}, \quad \tan\theta = \frac{y}{x}.$$



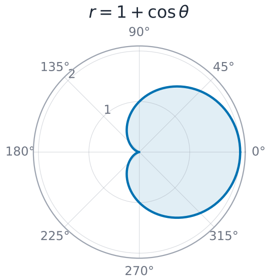



## Polar graphs and their rates of change

A polar function $r = f(\theta)$ sweeps a curve as $\theta$ turns.

> $r$ increasing means the curve moves away from the origin; $r$ decreasing, toward it. A negative $r$ plots opposite the angle.

$r = 1 + \cos\theta$




# 5 Recap

## Unit 3 – key takeaways

**1 The unit circle**

$\cos = x/r$, $\sin = y/r$, $\tan =$ the ray's slope

**2 Sinusoids**

$a\sin(b(\theta + c)) + d$: amplitude, period, shift, midline

**3 Equations**

infinitely many solutions – add the period

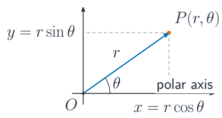
**4 Polar**

$x = r\cos\theta$, $y = r\sin\theta$; $r$ rising moves away from O



## Check yourself

1. Give the amplitude, period and midline of $3\sin(2\theta) + 1$.
2. Solve $2\sin\theta = 1$ on $[0, 2\pi)$.
3. Why does $\arcsin$ need a restricted domain?
4. Convert: what are $x$ and $y$ in terms of $r$ and $\theta$?




**Answers** **1.** $3$, $\pi$, $y = 1$ **2.** $\frac{\pi}{6}$, $\frac{5\pi}{6}$ **3.** sine is not one-to-one **4.** $x = r\cos\theta$, $y = r\sin\theta$