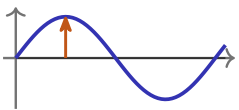
Trigonometric & Polar Functions

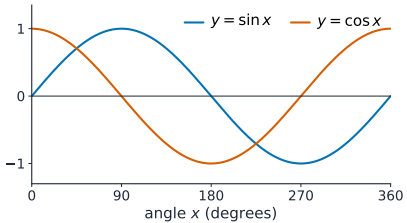
AP Precalculus ▪ Unit 3

AP Precalculus



What we will cover

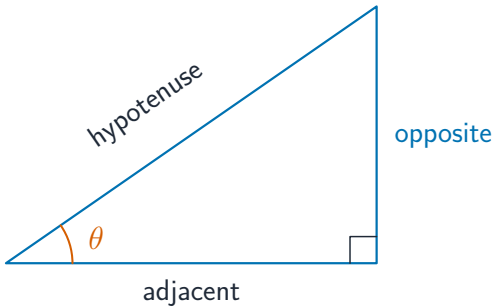
1. Periodic phenomena
2. Sine cosine & tangent
3. Sinusoidal functions
4. Transformations & modelling
5. Inverse trig & equations
6. Identities
7. Polar functions



Periodic phenomena

A **periodic** 周期 function repeats its output values over equal input intervals – tides, daylight hours, a rotating wheel.

> Period and **frequency** 频率 are reciprocals. $\sin\theta$ has period 2π.



Sine, cosine and tangent on the unit circle

For a point (x, y) at distance r along the terminal ray:

$$\cos\theta = \frac{x}{r}, \quad \sin\theta = \frac{y}{r}, \quad \tan\theta = \frac{y}{x}.$$

> $\tan\theta$ is the slope of the terminal ray – which is why it is undefined at $\theta = \frac{\pi}{2}$.

ferris

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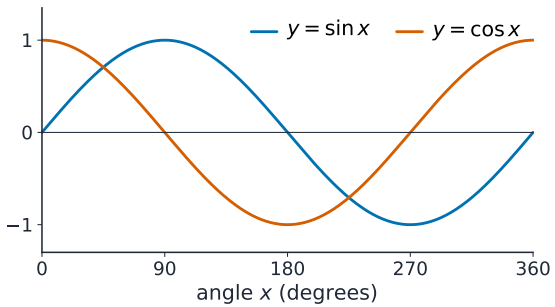
Amplitude, midline and period

- **Amplitude** 振幅 = half the distance between max and min
- **Midline** 中线 $y = d$ = the average of max and min
- Period $= \dfrac{2\pi}{b}$



Worked example – read a sinusoid

$f(\theta) = 3\sin(2\theta) + 1$

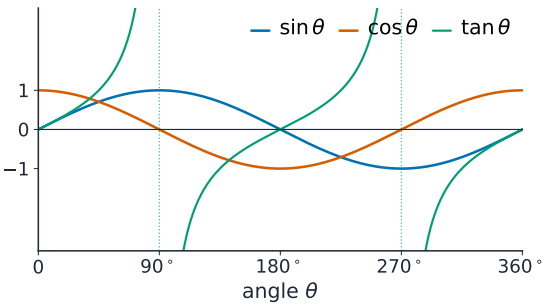
Amplitude 3; period $\dfrac{2\pi}{2} = \pi$; midline $y = 1$. So the maximum is $1 + 3 = 4$ and the minimum is $1 - 3 = -2$.

The general form is

$$a\sin\big(b(\theta + c)\big) + d.$$

a sets amplitude, b the period, $-c$ the phase shift (a $+c$ shifts left), d the midline.

Transformations and data modelling

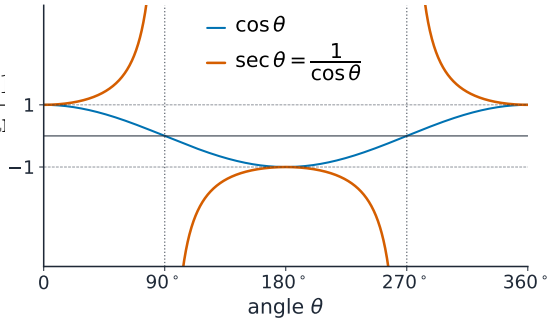


polar Orbit

Tangent, and the reciprocal functions

$$\sec\theta = \frac{1}{\cos\theta}, \quad \csc\theta = \frac{1}{\sin\theta}, \quad \cot\theta = \frac{1}{\tan}$$

$\tan\theta$ has period π (not 2π) and a vertical asymptote wherever $\cos\theta = 0$.



Inverse trigonometric functions

Sine, cosine and tangent are not one-to-one, so each inverse needs a restricted domain.

> An inverse trig function returns an angle – and only the one in its restricted range.

arcsin: range $[-\frac{\pi}{2}, \frac{\pi}{2}]$

arccos: range $[0, \pi]$

arctan: range $(-\frac{\pi}{2}, \frac{\pi}{2})$

Worked example – a trigonometric equation

Solve $2\sin\theta = 1$ on $[0, 2\pi)$

$\sin\theta = \frac{1}{2}$. On the unit circle sine is positive in the first and second quadrants:

$$\theta = \frac{\pi}{6} \quad \text{or} \quad \theta = \frac{5\pi}{6}.$$

Over all reals, add $2\pi k$ to each.

A trig equation has infinitely many solutions – the calculator gives you one. Add the period, and find the second quadrant solution yourself.

Equivalent representations – the identities

Pythagorean: $\sin^2\theta + \cos^2\theta = 1$

Sum: $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta \qquad \cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

Double angle: $\sin 2\theta = 2\sin\theta\cos\theta$

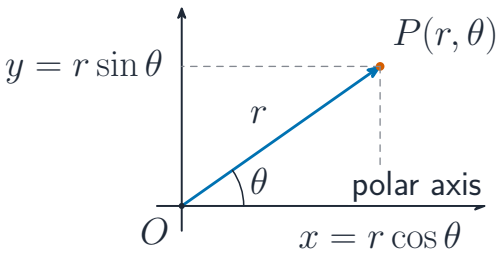
An **identity** 恒等式 is true for every θ – use one to rewrite an expression into a form you can solve.

Polar coordinates

Polar coordinates 极坐标 give a point by distance r and angle θ:

$$x = r\cos\theta, \quad y = r\sin\theta,$$

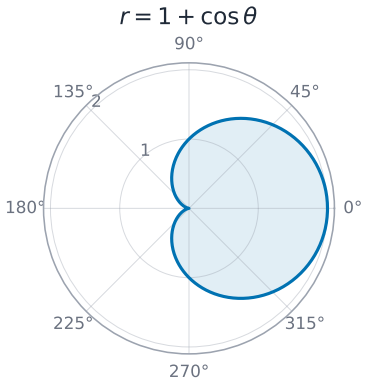
$$r = \sqrt{x^2 + y^2}, \quad \tan\theta = \frac{y}{x}.$$



Polar graphs and their rates of change

A polar function $r = f(\theta)$ sweeps a curve as θ turns.

> r increasing means the curve moves away from the origin; r decreasing, toward it. A negative r plots opposite the angle.



Unit 3 – key takeaways

1 The unit circle

$\cos = x/r$, $\sin = y/r$, $\tan =$ the ray's slope

2 Sinusoids

$a\sin(b(\theta + c)) + d$: amplitude, period, shift, midline

3 Equations

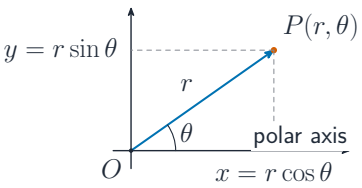
infinitely many solutions – add the period

4 Polar

$x = r\cos\theta$, $y = r\sin\theta$; r rising moves away from O

Check yourself

1. Give the amplitude, period and midline of $3\sin(2\theta) + 1$.
2. Solve $2\sin\theta = 1$ on $[0, 2\pi)$.
3. Why does $\arcsin$ need a restricted domain?
4. Convert: what are x and y in terms of r and θ?



Answers **1.** 3, π, $y = 1$ **2.** $\frac{\pi}{6}$, $\frac{5\pi}{6}$ **3.** sine is not one-to-one **4.** $x = r\cos\theta$, $y = r\sin\theta$