AP Precalculus

Learn these by heart before the exam. 考试前把这些内容背熟。

Polynomial and Rational Functions

Rate of change: a polynomial of degree n has a constant nth DIFFERENCE over equal input steps. Linear has constant first differences; quadratic constant second.

End behaviour is set by the leading term. Even degree: both ends agree. Odd degree: ends disagree. Negative leading coefficient flips both.

Zero of multiplicity: odd multiplicity CROSSES the x-axis; even multiplicity touches and turns.

$f(x) = \frac{p(x)}{q(x)}$ vertical asymptote where $q = 0$ and $p \neq 0$; a HOLE where both are 0 有理函数

Horizontal asymptote: $\deg p < \deg q$ *gives* $y = 0$*; equal degrees give the ratio of leading coefficients;* $\deg p = \deg q + 1$ *gives a SLANT asymptote (divide).*

$y = af(b(x-h)) + k$ h and k shift; a scales vertically; b scales horizontally by $1/b$ 函数变换

Even $f(-x) = f(x)$ (symmetric in the y-axis). Odd $f(-x) = -f(x)$ (symmetric about the origin).

Exponential and Logarithmic Functions

Exponential has a constant RATIO over equal input steps; linear has a constant difference. That is how you tell them apart from a table.

$f(x) = ab^x, \quad f(x) = ae^{kx}$ growth if $b > 1$ or $k > 0$; decay if $0 < b < 1$ or $k < 0$ 指数函数

$y = \log_b x \iff b^y = x$ 对数的定义

$\log(mn) = \log m + \log n, \quad \log \frac{m}{n} = \log m - \log n, \quad \log m^p = p \log m$ 对数运算法则

$\log_b x = \frac{\ln x}{\ln b}$ 换底公式

$A = P\left(1 + \frac{r}{n}\right)^{nt}, \quad A = Pe^{rt}$ 复利

Semi-log plot: exponential data becomes a STRAIGHT line when $\log y$ is plotted against x.

Log and exponential are inverses; reflect one in $y = x$ to get the other. Domain of $\log$ is $x > 0$.

Trigonometric and Polar Functions

$s = r\theta, \quad 180° = \pi$ rad 弧长与弧度

$\sin\theta = \frac{y}{r}, \quad \cos\theta = \frac{x}{r}, \quad \tan\theta = \frac{y}{x}$ 单位圆定义

$\sin^2\theta + \cos^2\theta = 1, \quad 1 + \tan^2\theta = \sec^2\theta$ 毕达哥拉斯恒等式

$y = a\sin(b(x-h)) + k$ amplitude $|a|$, period $\frac{2\pi}{|b|}$, phase shift h, midline $y = k$ 正弦型函数

$\sin(A \pm B) = \sin A\cos B \pm \cos A \sin B$ 和差公式

$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B, \quad \sin 2A = 2\sin A\cos A$ 和差与倍角

$\frac{a}{\sin A} = \frac{b}{\sin B}, \quad c^2 = a^2 + b^2 - 2ab\cos C$ 正弦定理与余弦定理

$x = r\cos\theta, \quad y = r\sin\theta, \quad r^2 = x^2 + y^2, \quad \tan\theta = \frac{y}{x}$ 极坐标转换

Polar graphs: $r = a$ circle, $r = a\cos\theta$ circle through the origin, $r = a \pm b\cos\theta$ limacon/cardioid, $r = a\cos(n\theta)$ rose.

r increasing as θ increases means the curve moves AWAY from the origin. Say it that way in a description.

Functions Involving Parameters, Vectors, and Matrices

$x = f(t), \quad y = g(t)$ eliminate t to get the Cartesian form; keep the direction of travel 参数方程

$\vec{v} = \langle a, b\rangle, \quad |\vec{v}| = \sqrt{a^2 + b^2}, \quad \theta = \arctan\frac{b}{a}$ 向量的大小与方向

$\vec{u} + \vec{v} = \langle u_1 + v_1,\ u_2 + v_2\rangle, \quad k\vec{v} = \langle ka, kb\rangle$ 向量加法与数乘

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} ax + by \\ cx + dy \end{bmatrix}$ 矩阵作为线性变换

$\det = ad - bc, \quad A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ det is the AREA scale factor; $\det = 0$ means no inverse 行列式与逆矩阵

Matrix multiplication is NOT commutative: $AB \neq BA$ in general. Order is the transformation order, right to left.

Exam technique

Describe behaviour in words AND in context: 'as x increases without bound, $f(x)$ approaches 3 from below'.

State the domain restriction whenever you invert, take a log, or divide. Examiners look for it.

Show which model fits and WHY: constant difference means linear, constant ratio means exponential, constant second difference means quadratic.