AP Precalculus

Formula Sheet · 公式表

No formula sheet is supplied, but this exam is less about formulae than about the behaviour of four function families and the ability to say what a change to a parameter does. This booklet lists what must be recalled, and – since most of the credit sits there – how to phrase the behaviour claims the rubric is looking for. 本科目不提供公式表，重点在于函数族的性质与规范表述。

Polynomial and rational functions

End behaviour 端行为 — Determined by the leading term alone. Even degree with a positive leading coefficient rises at both ends; odd degree with a positive coefficient falls left and rises right.

Multiplicity 重根 — An odd multiplicity crosses the x-axis; an even multiplicity touches and turns back. Stating this is a standard scored justification.

Rates of change 变化率 — Average rate of change over $[a, b]$ is $\frac{f(b)-f(a)}{b-a}$. A function is concave up where the average rates of change are increasing – the exam phrases it exactly this way.

Asymptotes 渐近线 — A vertical asymptote comes from a zero of the denominator that does not cancel; a cancelling factor gives a HOLE instead. Horizontal behaviour comes from comparing the degrees, and equal degrees give the ratio of leading coefficients.

Exponential and logarithmic functions

The defining property 定义性质 — An exponential function changes by a constant FACTOR over equal intervals; a linear function changes by a constant amount. Identifying which from a table is a recurring first question.

General form 一般形式 — $f(x) = ab^x$, with a the initial value and b the growth factor. A growth rate of 7 per cent means $b = 1.07$; a decay of 7 per cent means $b = 0.93$.

Logarithm laws 对数运算律 — $\log(xy) = \log x + \log y$; $\log(x/y) = \log x - \log y$; $\log x^n = n \log x$; change of base $\log_b x = \frac{\ln x}{\ln b}$.

Inverse relationship 反函数关系 — $b^{\log_b x} = x$ and $\log_b(b^x) = x$. Solving an exponential equation means taking a logarithm of both sides; solving a logarithmic one means exponentiating – and then checking for extraneous solutions, since the domain excludes non-positive arguments.

Semi-log plots 半对数图 — Exponential data plotted with a logarithmic vertical axis becomes a straight line, and the slope of that line is $\log b$. This appears every year.

Trigonometric and polar functions

The unit circle 单位圆 — Sine is the y-coordinate, cosine the x-coordinate. The exact values at multiples of $\pi/6$ and $\pi/4$ must be known without a calculator.

Sinusoidal form 正弦型函数 — $f(x) = a \sin(b(x-h)) + k$: amplitude $|a|$, period $2\pi/|b|$, horizontal shift h, midline $y = k$. Reading these four off a graph, and building the equation back from them, is the core skill.

Identities to recall 必背恒等式 — $\sin^2\theta + \cos^2\theta = 1$; $\tan\theta = \sin\theta/\cos\theta$; the double-angle forms $\sin 2\theta = 2 \sin\theta \cos\theta$ and $\cos 2\theta = \cos^2\theta - \sin^2\theta$.

Inverse trigonometric ranges 反三角函数值域 — Arcsine returns values in $[-\pi/2, \pi/2]$ and arccosine in $[0, \pi]$. A calculator gives one solution; the question usually wants all of them in an interval, so use the symmetry of the circle.

Polar 极坐标 — $x = r\cos\theta$, $y = r\sin\theta$, $r^2 = x^2 + y^2$. Describe a polar graph by whether r is increasing or decreasing as θ increases, and note that a negative r plots opposite the angle.

Transformations and composition

Order of operations 变换顺序 — In $af(b(x-h)) + k$, the inside operations act on x and run opposite to intuition: b compresses horizontally by a factor $1/b$, and h shifts right. The outside operations act on y and behave as expected.

Inverse functions 反函数 — Swap x and y and solve. The domain of the inverse is the range of the original, and a function needs to be one-to-one before an inverse exists – say so when asked to justify.

Composition 复合函数 — The domain of $f(g(x))$ excludes any x outside the domain of g, and any x whose $g(x)$ falls outside the domain of f. The second condition is the one that is forgotten.

Phrasing that earns the point

Describe behaviour with both a direction and a rate: “as x increases, f increases at a decreasing rate”. A bare “it goes up” does not score.

When constructing a model from a description, define your variables including their units and state the domain over which the model is reasonable. Both are scored, separately from the equation.

Justify a choice of model by the property in the data: constant differences mean linear, constant ratios mean exponential, a repeating pattern with a fixed period means sinusoidal.