Question 1: Function Concepts
Part A: Graphing calculator required — 6 points

The function f is decreasing and is defined for all real numbers. The table gives values for $f(x)$ at selected values of x.

x	–2	–1	0	1	2
$f(x)$	14	7	3.5	1.75	0.875

The function g is given by $g(x) = -0.167x^3 + x^2 - 1.834$.

A

Model Solution

(i) The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(1)$ as a decimal approximation or indicate that it is not defined. Show the work that leads to your answer.

(ii) Find the value of $f^{-1}(3.5)$, or indicate that it is not defined.

Scoring		
(i) $h(1) = g(f(1)) = g(1.75) = 0.333$	Value	**Point A1**
(ii) From the table, $f^{-1}(3.5) = 0$.	Value	**Point A2**

General Scoring Notes for Question 1 Parts A, B, and C

- Decimal approximations must be accurate to three places after the decimal point by rounding or truncating. Decimal values of 0 in final digits need not be reported ($2.000 = 2.00 = 2.0 = 2$).
- A **decimal presentation error** occurs when a response is complete and correct, but the answer is reported to fewer digits than required.
- The first decimal presentation error in Question 1 does not earn the point. For each additional part of Question 1 that requires a decimal approximation and contains a decimal presentation error, the response is eligible to earn the point.

Scoring Notes for Part A

- **Point A1** is earned for a correct decimal approximation of 0.333 with supporting work of "$f(1)$" OR "$g(1.75)$" OR "1.75."
- **Point A2** does not require supporting work. **Point A2** is earned with a response of "0."

Partial Credit for Part A

A response that **does not** earn either **Point A1** or **Point A2** is eligible for **partial credit** in part A if the response has one criteria from the first column AND one criteria from the second column.

Partial credit response is scored **0** for **Point A1** and **1** for **Point A2**.

First Column	Second Column
Correct value in part A (i) without supporting work.	$f^{-1}(3.5)$ exists in part A (ii) without giving specific value.
Correct value in part A (i) that is not expressed as a decimal approximation.	
Correct value in part A (i) with a decimal presentation error.	

B

(i) Find all values of x, as decimal approximations, for which $g(x) = 0$, or indicate that there are no such values.

(ii) Determine the end behavior of g as x increases without bound. Express your answer using the mathematical notation of a limit.

(i) $g(x) = 0 \Rightarrow -0.167x^3 + x^2 - 1.834 = 0$ $x = -1.233$, $x = 1.578$, $x = 5.643$	Values	**Point B1**
(ii) As x increases without bound, the output values of g decrease without bound. Therefore, $\lim_{x\to\infty} g(x) = -\infty$.	End behavior with limit notation	**Point B2**

Scoring Notes for Part B

- **Point B1** does not require supporting work. **Point B1** is earned with the three correct decimal approximations -1.233, 1.578, and 5.643. The use of "$x =$" is not required.
- **Point B2** requires a correct limit statement with four components: "lim", "$x \to \infty$", the function g, and $-\infty$. Examples that earn **Point B2** include:
  - $\lim_{x\to\infty} g(x) = -\infty$ OR $\lim_{x\to\infty} g = -\infty$
  - $\lim_{x\to\infty} g(x) \to -\infty$ OR $\lim_{x\to\infty} g \to -\infty$
  - $\lim_{x\to\infty} g(x)$ $-\infty$ OR $\lim_{x\to\infty} g$ $-\infty$
- If the response includes an additional, complete limit statement (e.g., $\lim_{x\to-\infty} g(x) = \infty$), the limit statement must be correct.

Partial Credit for Part B

A response that **does not** earn either **Point B1** or **Point B2** is eligible for **partial credit** in part B if the response has one criteria from the first column AND one criteria from the second column.

Partial credit response is scored **1** for **Point B1** and **0** for **Point B2**.

First Column	Second Column
Three correct values in part B (i) with values that are not expressed as decimal approximations	Correct end behavior statement in part B (ii) without use of limit notation
Three correct values in part B (i) with a decimal presentation error	Correct end behavior statement in part B (ii) with incorrect limit notation
Only one correct value in part B (i) with no incorrect values included (may have a decimal presentation error)	Correct limit statement in part B (ii) that is missing "$x \to \infty$"
Only two correct values in part B (i) with no incorrect values included (may have a decimal presentation error)	

C **(i)** Based on the table, which of the following function types best models function f: linear, quadratic, exponential, or logarithmic?

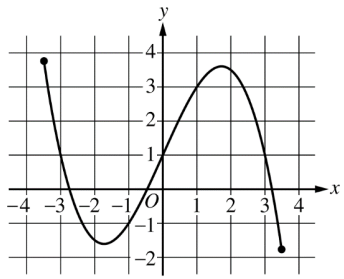
(ii) Give a reason for your answer in part C (i) based on the relationship between the change in the output values of f and the change in the input values of f. Refer to the values in the table in your reasoning.

(i) An exponential function best models f.	Answer	**Point C1**
(ii) In this case, the input-value intervals all have length 1. Examining the ratios of successive output values gives $\frac{f(-1)}{f(-2)} = \frac{f(0)}{f(-1)} = \frac{f(1)}{f(0)} = \frac{f(2)}{f(1)} = 0.5$. Because the successive output values over equal-length input-value intervals are proportional, an exponential model is best.	Reasoning	**Point C2**

Scoring Notes for Part C

- **Point C1** is earned for a correct function model with no incorrect function models listed. A response of "exponential" earns **Point C1**.
- Both **Point C1** and **Point C2** may be earned in part C (ii) provided there is no incorrect response in part C (i).
- **Point C2** requires an implicit or explicit reference to output values and input values from the table as support for the reason. For example, $\frac{f(n+1)}{f(n)} = 0.5$ OR "successive output values are decreasing proportionately by a factor of 0.5." The reasoning must demonstrate that the ratio of 0.5 applies to more than one pair of successive output values.
- A reason that references "exponential regression," "r values," OR "r^2 values" is not sufficient to earn **Point C2**.
- Special case: A response that indicates that f is best modeled by a **linear**, **quadratic**, or **logarithmic** function in part C (i) without a reason in part C (i) combined with a response in part C (ii) that provides both the correct answer and a correct reason is scored **0** for **Point C1** and **1** for **Point C2**.

Question 1: Function Concepts
Part A: Graphing calculator required — 6 points



Graph of f

The figure shows the graph of the function f on its domain of $-3.5 \le x \le 3.5$. The points $(-3, 1)$, $(0, 1)$, and $(3, 1)$ are on the graph of f. The function g is given by $g(x) = 2.916 \cdot (0.7)^x$.

(A)

Model Solution

(i) The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(3)$ as a decimal approximation, or indicate that it is not defined.

(ii) Find all values of x for which $f(x) = 1$, or indicate that there are no such values.

Scoring

(i) $h(3) = g(f(3)) = g(1) = 2.041$	Value	**1 point**
(ii) From the graph, $f(x) = 1$ when $x = -3$, $x = 0$, and $x = 3$.	Values	**1 point**

General Scoring Notes for Question 1 Parts (A), (B), and (C):

- Decimal approximations must be correct to three places after the decimal point by rounding or truncating. Decimal values of 0 in final digits need not be reported ($2.000 = 2.00 = 2.0 = 2$).
- A **decimal presentation error** occurs when a response is complete and correct, but the answer is reported to fewer digits than required.
- The first decimal presentation error in Question 1 does not earn the point. For each additional part of Question 1 that requires a decimal approximation and contains a decimal presentation error, the response is eligible to earn the point.

Scoring notes:

- The first point is earned for a correct decimal approximation of 2.041.
- The second point does not require supporting work.

- A response that does not earn either point in Part (A) is eligible for **partial credit** in Part (A) if the response has one criteria from the first column AND one criteria from the second column. Partial credit response is scored **0-1** in Part (A).

First Column	Second Column
Correct value in (i) that is not expressed as a decimal approximation	Only one correct value in (ii) with no incorrect values included
Correct value in (i) with a decimal presentation error	Only two correct values in (ii) with no incorrect values included

(B) (i) Find all values of x, as decimal approximations, for which $g(x) = 2$, or indicate that there are no such values.
(ii) Determine the end behavior of g as x increases without bound. Express your answer using the mathematical notation of a limit.

(i) $g(x) = 2 \Rightarrow 2.916(0.7)^x = 2$ $x = 1.057$	Value	**1 point**
(ii) As x increases without bound, the output values of g get arbitrarily close to 0. Therefore, $\lim_{x\to\infty} g(x) = 0$.	End behavior with limit notation	**1 point**

Scoring notes:

- The first point is earned for a correct decimal approximation of 1.057. No incorrect values may be included.
- The second point requires a correct limit statement with four components: "lim," "$x \to \infty$," the function g, and 0. Examples that earn the point include:
  - $\lim_{x\to\infty} g(x) = 0$ OR $\lim_{x\to\infty} g = 0$
  - $\lim_{x\to\infty} g(x) \to 0$ OR $\lim_{x\to\infty} g \to 0$
  - $\lim_{x\to\infty} g(x)$ 0 OR $\lim_{x\to\infty} g$ 0

  If the response includes an additional, complete limit statement (e.g., $\lim_{x\to-\infty} g(x) = \infty$), the value of the limit must be correct.

- A response that does not earn either point in Part (B) is eligible for **partial credit** in Part (B) if the response has one criteria from the first column AND one criteria from the second column. Partial credit response is scored **1-0** in Part (B).

First Column	Second Column
Correct answer in (i) that is not expressed as a decimal approximation	Correct end behavior statement in (ii) without use of limit notation
Correct value in (i) with a decimal presentation error	Correct end behavior statement in (ii) with incorrect limit notation
	Correct limit statement in (ii) that is missing "$x \to \infty$"

(C) (i) Determine if f has an inverse function.
(ii) Give a reason for your answer based on the definition of a function and the graph of $y = f(x)$.

(i) f does not have an inverse function on its domain of $-3.5 \le x \le 3.5$.	Answer	**1 point**
(ii) There are output values of f that are not mapped from unique input values; for example, $f(-3) = f(0) = f(3) = 1$.	Reason	**1 point**

Scoring notes:

- The first point is earned for a correct answer.
- Both points may be earned in (ii) provided there is no incorrect response in (i).
- The second point requires an implicit or explicit reference to the definition of a function AND support for the reason by referencing specific function values.
- A response such as "f does not have an inverse function because $f(-3) = f(0) = 1$" OR "f does not have an inverse function because there are two input values mapped to 1" earns both points.
- A response such as "f is not one-to-one" OR "f fails the horizontal line test" OR "There are output values that are not mapped from unique input values" is not sufficient to earn the second point.
- The second point cannot be earned if there are any errors in Part (C) (ii).
- A response that indicates that f **has** an inverse function in Part (C) (i) without a reason in Part (C) (i) combined with a response in Part (C) (ii) that provides both the correct answer and a correct reason is scored **0-1**.

Total for question 1 **6 points**