2. A person purchased a car at the end of the year 2019 ($t = 0$). The car's value decreases over time. At the end of the year 2020 ($t = 1$), the car was valued at 27.2 thousand dollars. At the end of the year 2025 ($t = 6$), the car was valued at 14.8 thousand dollars.

The value of the car can be modeled by the function V given by $V(t) = ab^t$, where $V(t)$ is the value of the car, in thousands of dollars, at time t, and t is the number of years since the end of 2019.

A.

i. Use the given data to write two equations that can be used to find the values for constants a and b in the expression for $V(t)$.

ii. Find the values for a and b as decimal approximations.

B.

i. Use the given data to find the average rate of change of the value of the car, in thousands of dollars per year, from $t = 1$ to $t = 6$ years. Express your answer as a decimal approximation. Show the computations that lead to your answer.

ii. Use the average rate of change found in part B (i) to estimate the value of the car, in thousands of dollars, at $t = 3$ years. Show the work that leads to your answer.

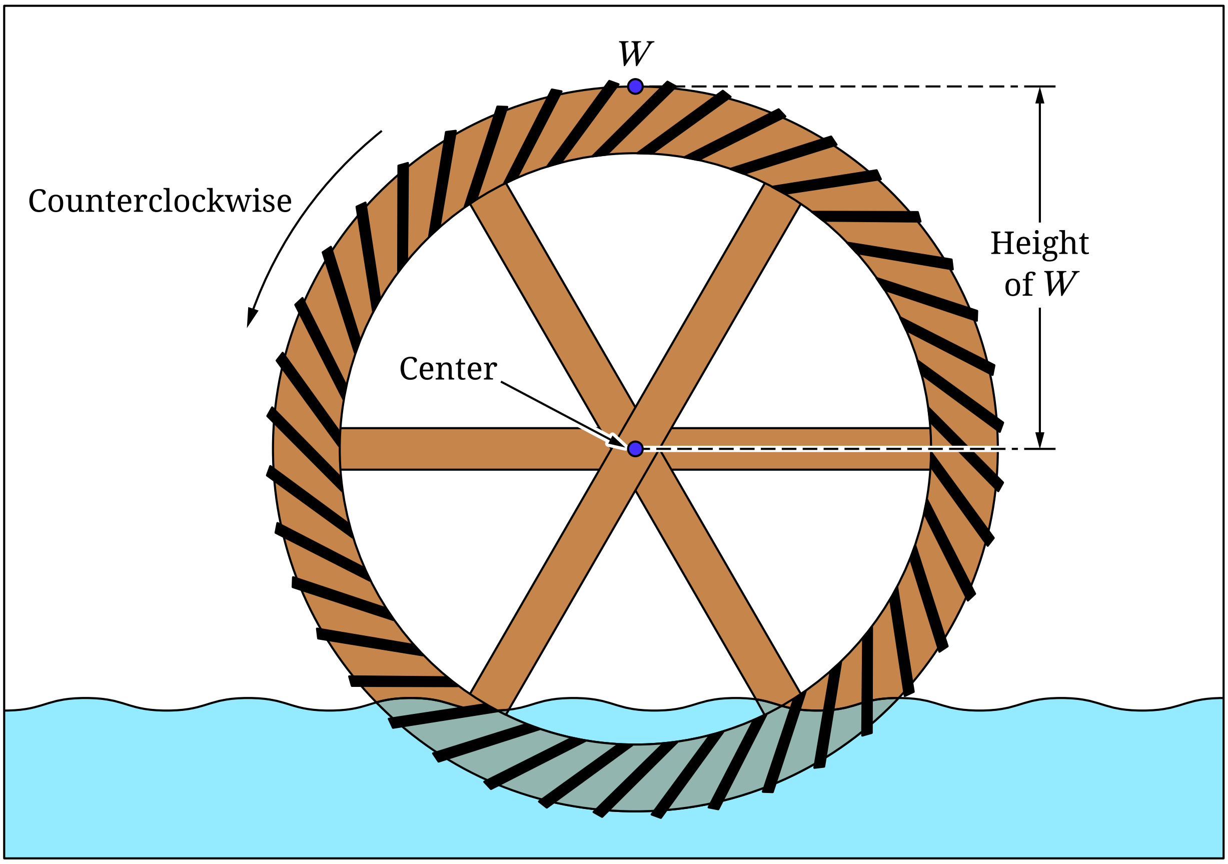
iii. The average rate of change found in part B (i) can be used to determine the secant line for the graph of V on the interval $1 \leq t \leq 6$. Let $A(t)$ represent the estimate of the value of the car, in thousands of dollars, at time t years, using the secant line. For $A(3)$, found in part B (ii), it can be shown that $A(3) > V(3)$.

In general, $A(t) > V(t)$ for all t, where $1 < t < 6$. Use the secant line and the graph of V to explain why this is true.

C. When the value of the car reaches 2 thousand dollars, the car's owner plans to donate it to an auto mechanic school. As a result, the car will immediately lose all of its value. Explain how this information can be used to determine a domain limitation for the model V.

END OF PART A

3. The figure shows a circular waterwheel that rotates in a counterclockwise direction at a constant rate and completes 1 revolution in 10 seconds. At time $t = 0$, point W on the edge of the waterwheel is at a height of 6 feet directly above the center of the waterwheel. The height of point W from the horizontal line through the center of the waterwheel periodically decreases and increases as the waterwheel moves.

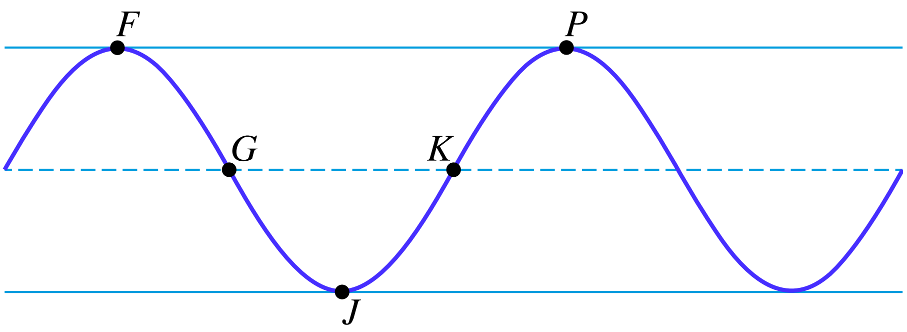


Note: Figure not drawn to scale.

The sinusoidal function h models the height of point W from the horizontal line through the center of the waterwheel, in feet, as a function of time t, in seconds. A positive value of $h(t)$ indicates W is above the line through the center; a negative value of $h(t)$ indicates W is below the line through the center.

A. The graph of h and its dashed midline for two full cycles is shown. Five points, F, G, J, K, and P, are labeled on the graph. No scale is indicated, and no axes are presented.

Determine possible coordinates $(t, h(t))$ for the five points: F, G, J, K, and P.



B. The function h can be written in the form $h(t) = a\sin(b(t+c)) + d$. Find values of constants a, b, c, and d.

C. Refer to the graph of h in part A. The t-coordinate of J is t_1, and the t-coordinate of K is t_2.

i. On the interval (t_1, t_2), which of the following is true about h?

a. h is positive and increasing.

b. h is positive and decreasing.

c. h is negative and increasing.

d. h is negative and decreasing.

ii. On the interval (t_1, t_2), describe the concavity of the graph of h and determine whether the rate of change of h is increasing or decreasing.

2. A musician released a new song on a streaming service. A streaming service is an online entertainment source that allows users to play music on their computers and mobile devices.

Several months later, the musician began using an app (at time $t = 0$) that counts the total number of plays for the song since its release. A "play" is a single stream of the song on the streaming service. The table gives the total number of plays, in thousands, for selected times t months after the musician began using the app. At $t = 0$, the total number of plays was 25 thousand. At $t = 2$, the total number of plays was 30 thousand. At $t = 4$, the total number of plays was 34 thousand.

Months after the musician began using the app	0	2	4
Total number of plays for the song since its release (thousands)	25	30	34

The total number of plays, in thousands, for the song since its release can be modeled by the function D given by $D(t) = at^2 + bt + c$, where $D(t)$ is the total number of plays, in thousands, for the song since its release, and t is the number of months after the musician began using the app.

A.

i. Use the given data to write three equations that can be used to find the values for constants a, b, and c in the expression for $D(t)$.

ii. Find the values for a, b, and c as decimal approximations.

B.

i. Use the given data to find the average rate of change of the total number of plays for the song, in thousands per month, from $t = 0$ to $t = 4$ months. Express your answer as a decimal approximation. Show the computations that lead to your answer.

ii. Use the average rate of change found in part B (i) to estimate the total number of plays for the song, in thousands, for $t = 1.5$ months. Show the work that leads to your answer.

iii. Let A_t represent the estimate of the total number of plays for the song, in thousands, using the average rate of change found in part B (i). For $A_{1.5}$ found in part B (ii), it can be shown that $A_{1.5} < D(1.5)$.
Explain why, in general, $A_t < D(t)$ for all t, where $0 < t < 4$. Your explanation should include a reference to the graph of D and its relationship to A_t.

C. The quadratic function model D has exactly one absolute minimum or one absolute maximum. That minimum or maximum can be used to determine a domain restriction for D.

Based on the context of the problem, explain how that minimum or maximum can be used to determine a boundary for the domain of D.

END OF PART A

2. On the initial day of sales ($t = 0$) for a new video game, there were 40 thousand units of the game sold that day. Ninety-one days later ($t = 91$), there were 76 thousand units of the game sold that day.

The number of units of the video game sold on a given day can be modeled by the function G given by $G(t) = a + b\ln(t + 1)$, where $G(t)$ is the number of units sold, in thousands, on day t since the initial day of sales.

(A) (i) Use the given data to write two equations that can be used to find the values for constants a and b in the expression for $G(t)$.

(ii) Find the values for a and b as decimal approximations.

(B) (i) Use the given data to find the average rate of change of the number of units of the video game sold, in thousands per day, from $t = 0$ to $t = 91$ days. Express your answer as a decimal approximation. Show the computations that lead to your answer.

(ii) Use the average rate of change found in (i) to estimate the number of units of the video game sold, in thousands, on day $t = 50$. Show the work that leads to your answer.

(iii) Let A_t represent the estimate of the number of units of the video game sold, in thousands, using the average rate of change found in (i). For A_{50}, found in (ii), it can be shown that $A_{50} < G(50)$. Explain why, in general, $A_t < G(t)$ for all t, where $0 < t < 91$.

(C) The makers of the video game reported that daily sales of the video game decreased each day after $t = 91$. Explain why the error in the model G increases after $t = 91$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

END OF PART A

PRECALCULUS

SECTION II, Part B

Time—30 minutes

2 Questions

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.