Question 1: Function Concepts
Part A: Graphing calculator required — 6 points

The function f is decreasing and is defined for all real numbers. The table gives values for $f(x)$ at selected values of x.

x	-2	-1	0	1	2
$f(x)$	14	7	3.5	1.75	0.875

The function g is given by $g(x) = -0.167x^3 + x^2 - 1.834$.

	Model Solution	Scoring	
A	**(i)** The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(1)$ as a decimal approximation or indicate that it is not defined. Show the work that leads to your answer. **(ii)** Find the value of $f^{-1}(3.5)$, or indicate that it is not defined.		
	(i) $h(1) = g(f(1)) = g(1.75) = 0.333$	Value	**Point A1**
	(ii) From the table, $f^{-1}(3.5) = 0$.	Value	**Point A2**

General Scoring Notes for Question 1 Parts A, B, and C

- Decimal approximations must be accurate to three places after the decimal point by rounding or truncating. Decimal values of 0 in final digits need not be reported ($2.000 = 2.00 = 2.0 = 2$).
- A **decimal presentation error** occurs when a response is complete and correct, but the answer is reported to fewer digits than required.
- The first decimal presentation error in Question 1 does not earn the point. For each additional part of Question 1 that requires a decimal approximation and contains a decimal presentation error, the response is eligible to earn the point.

Scoring Notes for Part A

- **Point A1** is earned for a correct decimal approximation of 0.333 with supporting work of "$f(1)$" OR "$g(1.75)$" OR "1.75."
- **Point A2** does not require supporting work. **Point A2** is earned with a response of "0."

Partial Credit for Part A

A response that **does not** earn either **Point A1** or **Point A2** is eligible for **partial credit** in part A if the response has one criteria from the first column AND one criteria from the second column.

Partial credit response is scored **0** for **Point A1** and **1** for **Point A2**.

First Column	**Second Column**
Correct value in part A (i) without supporting work.	$f^{-1}(3.5)$ exists in part A (ii) without giving specific value.
Correct value in part A (i) that is not expressed as a decimal approximation.	
Correct value in part A (i) with a decimal presentation error.	

B

(i) Find all values of x, as decimal approximations, for which $g(x) = 0$, or indicate that there are no such values.

(ii) Determine the end behavior of g as x increases without bound. Express your answer using the mathematical notation of a limit.

(i) $g(x) = 0 \Rightarrow -0.167x^3 + x^2 - 1.834 = 0$ $x = -1.233$, $x = 1.578$, $x = 5.643$	Values	**Point B1**
(ii) As x increases without bound, the output values of g decrease without bound. Therefore, $\lim\limits_{x \to \infty} g(x) = -\infty$.	End behavior with limit notation	**Point B2**

Scoring Notes for Part B

- **Point B1** does not require supporting work. **Point B1** is earned with the three correct decimal approximations -1.233, 1.578, and 5.643. The use of "$x =$" is not required.
- **Point B2** requires a correct limit statement with four components: "lim", "$x \to \infty$", the function g, and $-\infty$. Examples that earn **Point B2** include:
  - $\lim\limits_{x \to \infty} g(x) = -\infty$ OR $\lim\limits_{x \to \infty} g = -\infty$
  - $\lim\limits_{x \to \infty} g(x) \to -\infty$ OR $\lim\limits_{x \to \infty} g \to -\infty$
  - $\lim\limits_{x \to \infty} g(x) \quad -\infty$ OR $\lim\limits_{x \to \infty} g \quad -\infty$
- If the response includes an additional, complete limit statement (e.g., $\lim\limits_{x \to -\infty} g(x) = \infty$), the limit statement must be correct.

Partial Credit for Part B

A response that **does not** earn either **Point B1** or **Point B2** is eligible for **partial credit** in part B if the response has one criteria from the first column AND one criteria from the second column.

Partial credit response is scored **1** for **Point B1** and **0** for **Point B2**.

First Column	**Second Column**
Three correct values in part B (i) with values that are not expressed as decimal approximations	Correct end behavior statement in part B (ii) without use of limit notation
Three correct values in part B (i) with a decimal presentation error	Correct end behavior statement in part B (ii) with incorrect limit notation
Only one correct value in part B (i) with no incorrect values included (may have a decimal presentation error)	Correct limit statement in part B (ii) that is missing "$x \to \infty$"
Only two correct values in part B (i) with no incorrect values included (may have a decimal presentation error)	

C **(i)** Based on the table, which of the following function types best models function *f*: linear, quadratic, exponential, or logarithmic?

(ii) Give a reason for your answer in part C (i) based on the relationship between the change in the output values of *f* and the change in the input values of *f*. Refer to the values in the table in your reasoning.

(i) An exponential function best models *f*.	Answer	**Point C1**
(ii) In this case, the input-value intervals all have length 1. Examining the ratios of successive output values gives $\frac{f(-1)}{f(-2)} = \frac{f(0)}{f(-1)} = \frac{f(1)}{f(0)} = \frac{f(2)}{f(1)} = 0.5$. Because the successive output values over equal-length input-value intervals are proportional, an exponential model is best.	Reasoning	**Point C2**

Scoring Notes for Part C

- **Point C1** is earned for a correct function model with no incorrect function models listed. A response of "exponential" earns **Point C1**.
- Both **Point C1** and **Point C2** may be earned in part C (ii) provided there is no incorrect response in part C (i).
- **Point C2** requires an implicit or explicit reference to output values and input values from the table as support for the reason. For example, $\frac{f(n+1)}{f(n)} = 0.5$ OR "successive output values are decreasing proportionately by a factor of 0.5." The reasoning must demonstrate that the ratio of 0.5 applies to more than one pair of successive output values.
- A reason that references "exponential regression," "*r* values," OR "r^2 values" is not sufficient to earn **Point C2**.
- Special case: A response that indicates that *f* is best modeled by a **linear**, **quadratic**, or **logarithmic** function in part C (i) without a reason in part C (i) <u>combined</u> with a response in part C (ii) that provides both the correct answer <u>and</u> a correct reason is scored **0** for **Point C1** and **1** for **Point C2**.

Question 2: Modeling a Non-Periodic Context
Part A: Graphing calculator required — 6 points

A musician released a new song on a streaming service. A streaming service is an online entertainment source that allows users to play music on their computers and mobile devices.

Several months later, the musician began using an app (at time $t = 0$) that counts the total number of plays for the song since its release. A "play" is a single stream of the song on the streaming service. The table gives the total number of plays, in thousands, for selected times t months after the musician began using the app.
At $t = 0$, the total number of plays was 25 thousand. At $t = 2$, the total number of plays was 30 thousand.
At $t = 4$, the total number of plays was 34 thousand.

Months after the musician began using the app	0	2	4
Total number of plays for the song since its release (thousands)	25	30	34

The total number of plays, in thousands, for the song since its release can be modeled by the function D given by $D(t) = at^2 + bt + c$, where $D(t)$ is the total number of plays, in thousands, for the song since its release, and t is the number of months after the musician began using the app.

	Model Solution	Scoring	
A	**(i)** Use the given data to write three equations that can be used to find the values for constants a, b, and c in the expression for $D(t)$. **(ii)** Find the values for a, b, and c as decimal approximations.		
	(i) Because $D(0) = 25$, $D(2) = 30$, and $D(4) = 34$, three equations to find a, b, and c are $a(0)^2 + b(0) + c = 25$ $a(2)^2 + b(2) + c = 30$ $a(4)^2 + b(4) + c = 34$	Three equations	**Point A1**
	(ii) $c = 25$ $\begin{array}{r} 4a + 2b = 5 \\ \underline{16a + 4b = 9} \end{array} \Rightarrow \begin{array}{r} 16a + 8b = 20 \\ \underline{16a + 4b = 9} \end{array} \Rightarrow 4b = 11$ $b = \frac{11}{4} = 2.75$ $a = -0.125$ $D(t) = -0.125t^2 + 2.75t + 25$	Values of a, b, and c	**Point A2**

General Scoring Notes for Question 2 Parts A, B, and C

- Decimal approximations must be accurate to three places after the decimal point by rounding or truncating. Decimal values of 0 in final digits need not be reported ($2.000 = 2.00 = 2.0 = 2$).
- A **decimal presentation error** occurs when a response is complete and correct, but the answer is reported to fewer digits than required.
- The first decimal presentation error in Question 2 does not earn the point. For each additional part of Question 2 that requires a decimal approximation and contains a decimal presentation error, the response is eligible to earn the point.
- Parts of Question 2 require decimal answers. The first response in Question 2 that is complete, correct, and uses exact values rather than decimal form does not earn the point. For each additional part of Question 2 that requires a decimal answer, a response that is complete, correct, and uses exact values rather than decimal form is eligible to earn the point.

Scoring Notes for Part A

- **Point A1** is earned for presenting three equations involving *a*, *b*, and *c* that use the given input-output pairs.
- **Point A2** is earned for correct values of *a*, *b*, and *c* **with or without** supporting work. If correct values are identified, work should be ignored.
- **Point A2** is earned for correct values of *a*, *b*, and *c* presented as either standalone values OR in an expression for $D(t)$.
- A response is eligible to earn both **Point A1** and **Point A2** with a correct translation to “thousands.” Use of 25,000, 30,000, and 34,000 results in values of $a = -125$, $b = 2750$, and $c = 25{,}000$.

Partial Credit for Part A

A response that **does not** earn either **Point A1** or **Point A2** is eligible for **partial credit** in part A if the response has two correct equations in the presence of three equations involving *a*, *b*, and *c* AND one correct value.

Partial credit response is scored **1** for **Point A1** and **0** for **Point A2**.

B **(i)** Use the given data to find the average rate of change of the total number of plays for the song, in thousands per month, from $t = 0$ to $t = 4$ months. Express your answer as a decimal approximation. Show the computations that lead to your answer.

(ii) Use the average rate of change found in part B (i) to estimate the total number of plays for the song, in thousands, for $t = 1.5$ months. Show the work that leads to your answer.

(i) $\frac{D(4) - D(0)}{4 - 0} = \frac{(34 - 25)}{4} = 2.25$ The average rate of change is 2.25 thousand plays per month.	Average rate of change	**Point B1**

(ii) Estimates using the average rate of change can be given by $y = 25 + 2.25t$. For $t = 1.5$, $y = 25 + 2.25 \cdot 1.5 = 28.375$. The estimate of the total number of plays for the song for $t = 1.5$ months is 28.375 thousand.	Estimate using average rate of change	**Point B2**

Scoring Notes for Part B (i)–(ii)

- **Point B1** and **Point B2** both require supporting work.
- **Point B1** is earned for a correct decimal approximation in the presence of a quotient with a difference that uses the given data values. In part B (i) units are not needed and are ignored if presented.
- Another form of the secant line, derived from $D(4) = 34$, is $y = 34 + 2.25(t - 4)$.
- Eligibility for **Point B2**:
    - If a response in part B (i) has a decimal presentation error OR if a response in part B (i) is incorrect:
        - The reported value in part B (i) as the average rate of change can be used to arrive at an estimate in part B (ii). To earn **Point B2**, the estimate in part B (ii) must be consistent with both the reported value in part B (i) and the endpoint used in the supporting work in part B (ii).
- The final number in part B (ii) may be reported as 28 thousand provided the supporting work has a correct decimal approximation for the estimate.
- If a response has a correct translation to "thousands":
    - The response is eligible to earn both **Point B1** and **Point B2**.
    - Use of 25,000 and 34,000 results in an answer of 2250 in part B (i).
    - If a response earned **Point B1** without a decimal presentation error, then an estimate of 28,375 earns **Point B2** in the presence of supporting work.
    - If a response in part B (i) has a decimal presentation error OR if a response in part B (i) is incorrect:
        - The reported value in part B (i) as the average rate of change can be used to arrive at an estimate in part B (ii). To earn **Point B2**, the estimate in part B (ii) must be consistent with both the reported value in part B (i) and the endpoint used in the supporting work in part B (ii).

Partial Credit for Part B (i)–(ii)

A response that **does not** earn either **Point B1** or **Point B2** is eligible for **partial credit** in part B if the response has one criteria from the first column AND one criteria from the second column.

Partial credit response is scored **1** for **Point B1** and **0** for **Point B2**.

First Column	Second Column
Correct quotient in part B (i) that uses the given data values that is not expressed as a decimal approximation	Correct estimate of 28.375 in part B (ii) that does not include supporting work
Correct quotient in part B (i) that uses the given data values and has a decimal presentation error	Correct or consistent supporting work in part B (ii) that does not provide an estimate
Correct average rate of change in part B (i) that does not include supporting work	

B **(iii)** Let A_t represent the estimate of the total number of plays for the song, in thousands, using the average rate of change found in part B (i). For $A_{1.5}$ found in part B (ii), it can be shown that $A_{1.5} < D(1.5)$.

Explain why, in general, $A_t < D(t)$ for all t, where $0 < t < 4$. Your explanation should include a reference to the graph of D and its relationship to A_t.

(iii) The estimate A_t is the y-coordinate of a point on the secant line that passes through $(0, D(0))$ and $(4, D(4))$. Because the graph of D is concave down on the interval $0 < t < 4$, this secant line is below the graph of D on the interval $0 < t < 4$. Therefore, the estimate using the average rate of change A_t is less than the value of $D(t)$ for all t on the interval $0 < t < 4$.	Explanation	**Point B3**

Scoring Notes for Part B (iii)

To earn **Point B3**, the explanation must include:

- The graph of D is concave down OR the rate of change of D is decreasing.
- A reference to the use of a secant line on $0 < t < 4$ OR the use of a linear function with reference to t-values 0 and 4 that provide the placement of the line.

C	The quadratic function model D has exactly one absolute minimum or one absolute maximum. That minimum or maximum can be used to determine a domain restriction for D. Based on the context of the problem, explain how that minimum or maximum can be used to determine a boundary for the domain of D.	
	The total number of plays for the song must be nonnegative and never decreasing. The quadratic function D has an absolute maximum at $t = 11$. The function D increases before $t = 11$ and decreases after $t = 11$. Because D is also positive for positive t-values less than $t = 11$, the domain of model D is an interval with right endpoint $t = 11$.	Explanation **Point C1**

Scoring Notes for Part C

- A response that focuses on the left endpoint of the domain is eligible for **Point C1**.
- To earn **Point C1**, the explanation must include one of the following:
  - The total number of plays cannot decrease AND the function model D decreases after the time it achieves its absolute maximum.
    - The location of the absolute maximum at $t = 11$ is not required but must be correct if included.
  - The total number of plays is nonnegative AND the left endpoint of the domain is the only value of t at which $D(t)$ changes from negative to positive.
    - $D(t)$ changes from negative to positive at $t = -6.916$. This value is not required, but if included it must be accurate to at least one place after the decimal point, rounded or truncated.

Question 2: Modeling a Non-Periodic Context
Part A: Graphing calculator required — 6 points

On the initial day of sales ($t = 0$) for a new video game, there were 40 thousand units of the game sold that day. Ninety-one days later ($t = 91$), there were 76 thousand units of the game sold that day.

The number of units of the video game sold on a given day can be modeled by the function G given by $G(t) = a + b\ln(t+1)$, where $G(t)$ is the number of units sold, in thousands, on day t since the initial day of sales.

Model Solution	Scoring	

(A) (i) Use the given data to write two equations that can be used to find the values for constants a and b in the expression for $G(t)$.

(ii) Find the values for a and b as decimal approximations.

(i) Because $G(0) = 40$ and $G(91) = 76$, two equations to find a and b are $a + b\ln(0+1) = 40$ $a + b\ln(91+1) = 76$.	Two equations	**1 point**
(ii) $a = 40 - b\ln 1 = 40$ $b = \dfrac{(76-40)}{\ln 92} = 7.961451$ $G(t) = 40 + 7.961\ln(t+1)$	Values of a and b	**1 point**

General Scoring Notes for Question 2 Parts (A), (B), and (C):

- Decimal approximations must be correct to three places after the decimal point by rounding or truncating. Decimal values of 0 in final digits need not be reported ($2.000 = 2.00 = 2.0 = 2$).
- A **decimal presentation error** occurs when a response is complete and correct, but the answer is reported to fewer digits than required.
- The first decimal presentation error in Question 2 does not earn the point. For each additional part of Question 2 that requires a decimal approximation and contains a decimal presentation error, the response is eligible to earn the point.

Scoring notes:

- The first point is earned for presenting two equations involving a and b that use the given input-output pairs.
- The second point is earned for correct values of a and b with or without supporting work. If correct values are identified, work should be ignored.
- The second point is earned for correct values of a and b presented as either stand-alone values OR in an expression for $G(t)$.
- A response is eligible to earn both points with a correct translation to "thousands." Use of $40{,}000$ and $76{,}000$ results in values of $a = 40{,}000$ and $b = 7961$.
- A response that does not earn either point in Part (A) is eligible for **partial credit** in Part (A) if the response has one correct equation in the presence of two equations involving a and b AND one correct value. Partial credit response is scored **1-0** in Part (A).

(B) (i) Use the given data to find the average rate of change of the number of units of the video game sold, in thousands per day, from $t = 0$ to $t = 91$ days. Express your answer as a decimal approximation. Show the computations that lead to your answer.

(ii) Use the average rate of change found in (i) to estimate the number of units of the video game sold, in thousands, on day $t = 50$. Show the work that leads to your answer.

(iii) Let A_t represent the estimate of the number of units of the video game sold, in thousands, using the average rate of change found in (i). For A_{50}, found in (ii), it can be shown that $A_{50} < G(50)$. Explain why, in general, $A_t < G(t)$ for all t, where $0 < t < 91$.

(i) $\dfrac{G(91) - G(0)}{91 - 0} = \dfrac{(76 - 40)}{91} = 0.395604$ The average rate of change is 0.396 (or 0.395) thousand units per day.	Average rate of change	**1 point**
(ii) The average rate of change is $r = \dfrac{G(91) - G(0)}{91 - 0} = 0.395604$. The secant line between point $(0, G(0))$ and point $(91, G(91))$ is given by $y = y_1 + r(x - x_1)$, where (x_1, y_1) can be either one of the points. Estimates using the average rate of change are given by $y = G(0) + r(t - 0)$ OR $y = G(91) + r(t - 91)$. Both of these produce the same estimate. For $t = 50$, $y = 40 + r(50 - 0) = 59.780$ OR $y = 76 + r(50 - 91) = 59.780$. The number of units sold on day $t = 50$ was approximately 59.780 thousand.	Estimate using average rate of change	**1 point**
(iii) The estimate A_t is the y-coordinate of a point on the secant line that passes through $(0, G(0))$ and $(91, G(91))$. Because the graph of G is concave down on the interval $(0, 91)$, this secant line is below the graph of G on the interval $(0, 91)$. Therefore, the estimate A_t is less than the value of $G(t)$ for all t on the interval $(0, 91)$.	Answer with explanation	**1 point**

Scoring notes:

- Supporting work is required in (i) and (ii).
- The first point is earned for a correct decimal approximation in the presence of a quotient that uses the given data values. Units are not needed and are ignored if presented.
- Eligibility for the second point:
    - If a response earned the point in (i) without a decimal presentation error, then an estimate in the range $[59.750, 59.805]$ earns the second point in the presence of supporting work.
    - If a response in (i) has a decimal presentation error, the reported value in (i) as the average rate of change can be used to arrive at an estimate in (ii). To earn the second point, the estimate in (ii) must be consistent with both the reported value in (i) and the endpoint used in the supporting work in (ii).
    - If a response in (i) is incorrect, the reported value in (i) as the average rate of change can be used to arrive at an estimate in (ii). To earn the second point, the estimate in (ii) must be consistent with both the reported value in (i) and the endpoint used in the supporting work in (ii).
- The final number in (ii) may be reported as 59 thousand or 60 thousand provided the supporting work has a correct decimal approximation for the estimate.
- A response is eligible to earn both points with a correct translation to "thousands."
    - Use of $40,000$ and $76,000$ results in an answer of 395.604 in (i).
    - If a response earned the point in (i) without a decimal presentation error, then an estimate in the range $[59,750, 59,805]$ earns the second point in the presence of supporting work.
    - If a response in (i) has a decimal presentation error, the reported value in (i) as the average rate of change can be used to arrive at an estimate in (ii). To earn the second point, the estimate in (ii) must be consistent with both the reported value in (i) and the endpoint used in the supporting work in (ii).
    - If a response in (i) is incorrect, the reported value in (i) as the average rate of change can be used to arrive at an estimate in (ii). To earn the second point, the estimate in (ii) must be consistent with both the reported value in (i) and the endpoint used in the supporting work in (ii).
- A response that does not earn either point in Part (B) (i) and Part (B) (ii) is eligible for **partial credit** in Part (B) if the response has one criteria from the first column AND one criteria from the second column. Partial credit response is scored **1-0** in Part (B)(i)/(ii).

First Column	**Second Column**
A correct quotient that uses the given data values that is not expressed as a decimal approximation	A correct estimate in (ii) that does not include supporting work
A correct quotient that uses the given data values and has a decimal presentation error	Correct supporting work in (ii) that does not provide an estimate
A correct average rate of change in (i) that does not include supporting work	

- To earn the third point, the reasoning must include:
  - The graph of G is concave down OR the rate of change of G is decreasing
  - A reference to the use of a secant line on $0 < t < 91$ OR the use of a linear function with reference to endpoints 0 and 91 that provide the placement of the line

(C) The makers of the video game reported that daily sales of the video game decreased each day after $t = 91$. Explain why the error in the model G increases after $t = 91$.

On day $t = 91$, the output for daily sales and $G(91)$ are the same. For $t > 91$, daily sales are decreasing and G is increasing. Therefore, the absolute value of the difference between the actual daily sales and the daily sales predicted by G is increasing each day for $t > 91$.	Answer with reason	**1 point**

Scoring notes:

- To earn the point, the reasoning must include an implicit or explicit connection between the "function model is increasing" and "daily sales are decreasing."

Total for question 2 **6 points**