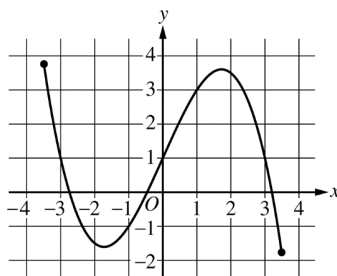


**Question 1: Function Concepts****Part A: Graphing calculator required****6 points**Graph of  $f$ 

The figure shows the graph of the function  $f$  on its domain of  $-3.5 \leq x \leq 3.5$ . The points  $(-3, 1)$ ,  $(0, 1)$ , and  $(3, 1)$  are on the graph of  $f$ . The function  $g$  is given by  $g(x) = 2.916 \cdot (0.7)^x$ .

**Model Solution****Scoring**

- (A) (i) The function  $h$  is defined by  $h(x) = (g \circ f)(x) = g(f(x))$ . Find the value of  $h(3)$  as a decimal approximation, or indicate that it is not defined.  
 (ii) Find all values of  $x$  for which  $f(x) = 1$ , or indicate that there are no such values.

|                                                                         |        |                |
|-------------------------------------------------------------------------|--------|----------------|
| (i) $h(3) = g(f(3)) = g(1) = 2.041$                                     | Value  | <b>1 point</b> |
| (ii) From the graph, $f(x) = 1$ when $x = -3$ , $x = 0$ , and $x = 3$ . | Values | <b>1 point</b> |

**General Scoring Notes for Question 1 Parts (A), (B), and (C):**

- Decimal approximations must be correct to three places after the decimal point by rounding or truncating. Decimal values of 0 in final digits need not be reported ( $2.000 = 2.00 = 2.0 = 2$ ).
- A **decimal presentation error** occurs when a response is complete and correct, but the answer is reported to fewer digits than required.
- The first decimal presentation error in Question 1 does not earn the point. For each additional part of Question 1 that requires a decimal approximation and contains a decimal presentation error, the response is eligible to earn the point.

**Scoring notes:**

- The first point is earned for a correct decimal approximation of 2.041.
- The second point does not require supporting work.

- A response that does not earn either point in Part (A) is eligible for **partial credit** in Part (A) if the response has one criteria from the first column AND one criteria from the second column. Partial credit response is scored **0-1** in Part (A).

| First Column                                                          | Second Column                                                     |
|-----------------------------------------------------------------------|-------------------------------------------------------------------|
| Correct value in (i) that is not expressed as a decimal approximation | Only one correct value in (ii) with no incorrect values included  |
| Correct value in (i) with a decimal presentation error                | Only two correct values in (ii) with no incorrect values included |

- (B) (i) Find all values of  $x$ , as decimal approximations, for which  $g(x) = 2$ , or indicate that there are no such values.  
 (ii) Determine the end behavior of  $g$  as  $x$  increases without bound. Express your answer using the mathematical notation of a limit.

|                                                                                                                                                  |                                  |                |
|--------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------|----------------|
| (i) $g(x) = 2 \Rightarrow 2.916(0.7)^x = 2$<br>$x = 1.057$                                                                                       | Value                            | <b>1 point</b> |
| (ii) As $x$ increases without bound, the output values of $g$ get arbitrarily close to 0.<br>Therefore, $\lim_{x \rightarrow \infty} g(x) = 0$ . | End behavior with limit notation | <b>1 point</b> |

**Scoring notes:**

- The first point is earned for a correct decimal approximation of 1.057. No incorrect values may be included.
- The second point requires a correct limit statement with four components: "lim," " $x \rightarrow \infty$ ," the function  $g$ , and 0. Examples that earn the point include:
  - $\lim_{x \rightarrow \infty} g(x) = 0$  OR  $\lim_{x \rightarrow \infty} g = 0$
  - $\lim_{x \rightarrow \infty} g(x) \rightarrow 0$  OR  $\lim_{x \rightarrow \infty} g \rightarrow 0$
  - $\lim_{x \rightarrow \infty} g(x) = 0$  OR  $\lim_{x \rightarrow \infty} g = 0$

If the response includes an additional, complete limit statement (e.g.,  $\lim_{x \rightarrow -\infty} g(x) = \infty$ ), the value of the limit must be correct.

- A response that does not earn either point in Part (B) is eligible for **partial credit** in Part (B) if the response has one criteria from the first column AND one criteria from the second column. Partial credit response is scored **1-0** in Part (B).

| First Column                                                           | Second Column                                                              |
|------------------------------------------------------------------------|----------------------------------------------------------------------------|
| Correct answer in (i) that is not expressed as a decimal approximation | Correct end behavior statement in (ii) without use of limit notation       |
| Correct value in (i) with a decimal presentation error                 | Correct end behavior statement in (ii) with incorrect limit notation       |
|                                                                        | Correct limit statement in (ii) that is missing " $x \rightarrow \infty$ " |

- (C) (i) Determine if  $f$  has an inverse function.  
(ii) Give a reason for your answer based on the definition of a function and the graph of  $y = f(x)$ .

|                                                                                                                            |        |                |
|----------------------------------------------------------------------------------------------------------------------------|--------|----------------|
| (i) $f$ does not have an inverse function on its domain of $-3.5 \leq x \leq 3.5$ .                                        | Answer | <b>1 point</b> |
| (ii) There are output values of $f$ that are not mapped from unique input values; for example, $f(-3) = f(0) = f(3) = 1$ . | Reason | <b>1 point</b> |

**Scoring notes:**

- The first point is earned for a correct answer.
- Both points may be earned in (ii) provided there is no incorrect response in (i).
- The second point requires an implicit or explicit reference to the definition of a function AND support for the reason by referencing specific function values.
- A response such as " $f$  does not have an inverse function because  $f(-3) = f(0) = 1$ " OR " $f$  does not have an inverse function because there are two input values mapped to 1" earns both points.
- A response such as " $f$  is not one-to-one" OR " $f$  fails the horizontal line test" OR "There are output values that are not mapped from unique input values" is not sufficient to earn the second point.
- The second point cannot be earned if there are any errors in Part (C) (ii).
- A response that indicates that  $f$  **has** an inverse function in Part (C) (i) without a reason in Part (C) (i) combined with a response in Part (C) (ii) that provides both the correct answer and a correct reason is scored **0-1**.

**Total for question 1      6 points**