

Question 1: Function Concepts
Part A: Graphing calculator required
6 points

The function f is decreasing and is defined for all real numbers. The table gives values for $f(x)$ at selected values of x .

x	-2	-1	0	1	2
$f(x)$	14	7	3.5	1.75	0.875

The function g is given by $g(x) = -0.167x^3 + x^2 - 1.834$.

	Model Solution	Scoring
A	(i) The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(1)$ as a decimal approximation or indicate that it is not defined. Show the work that leads to your answer.	
	(ii) Find the value of $f^{-1}(3.5)$, or indicate that it is not defined.	
	(i) $h(1) = g(f(1)) = g(1.75) = 0.333$	Value Point A1
	(ii) From the table, $f^{-1}(3.5) = 0$.	Value Point A2

General Scoring Notes for Question 1 Parts A, B, and C

- Decimal approximations must be accurate to three places after the decimal point by rounding or truncating. Decimal values of 0 in final digits need not be reported ($2.000 = 2.00 = 2.0 = 2$).
- A **decimal presentation error** occurs when a response is complete and correct, but the answer is reported to fewer digits than required.
- The first decimal presentation error in Question 1 does not earn the point. For each additional part of Question 1 that requires a decimal approximation and contains a decimal presentation error, the response is eligible to earn the point.

Scoring Notes for Part A

- Point A1** is earned for a correct decimal approximation of 0.333 with supporting work of “ $f(1)$ ” OR “ $g(1.75)$ ” OR “1.75.”
- Point A2** does not require supporting work. **Point A2** is earned with a response of “0.”

Partial Credit for Part A

A response that **does not** earn either **Point A1** or **Point A2** is eligible for **partial credit** in part A if the response has one criteria from the first column AND one criteria from the second column.

Partial credit response is scored **0** for **Point A1** and **1** for **Point A2**.

First Column	Second Column
Correct value in part A (i) without supporting work.	$f^{-1}(3.5)$ exists in part A (ii) without giving specific value.
Correct value in part A (i) that is not expressed as a decimal approximation.	
Correct value in part A (i) with a decimal presentation error.	

- B**
- (i) Find all values of x , as decimal approximations, for which $g(x) = 0$, or indicate that there are no such values.
- (ii) Determine the end behavior of g as x increases without bound. Express your answer using the mathematical notation of a limit.

(i) $g(x) = 0 \Rightarrow -0.167x^3 + x^2 - 1.834 = 0$ $x = -1.233, x = 1.578, x = 5.643$	Values Point B1
(ii) As x increases without bound, the output values of g decrease without bound. Therefore, $\lim_{x \rightarrow \infty} g(x) = -\infty$.	End behavior with limit notation Point B2

Scoring Notes for Part B

- Point B1** does not require supporting work. **Point B1** is earned with the three correct decimal approximations -1.233 , 1.578 , and 5.643 . The use of “ $x =$ ” is not required.
- Point B2** requires a correct limit statement with four components: “lim”, “ $x \rightarrow \infty$ ”, the function g , and $-\infty$. Examples that earn **Point B2** include:
 - $\lim_{x \rightarrow \infty} g(x) = -\infty$ OR $\lim_{x \rightarrow \infty} g = -\infty$
 - $\lim_{x \rightarrow \infty} g(x) \rightarrow -\infty$ OR $\lim_{x \rightarrow \infty} g \rightarrow -\infty$
 - $\lim_{x \rightarrow \infty} g(x) \quad -\infty$ OR $\lim_{x \rightarrow \infty} g \quad -\infty$
- If the response includes an additional, complete limit statement (e.g., $\lim_{x \rightarrow -\infty} g(x) = \infty$), the limit statement must be correct.

Partial Credit for Part B

A response that **does not** earn either **Point B1** or **Point B2** is eligible for **partial credit** in part B if the response has one criteria from the first column AND one criteria from the second column.

Partial credit response is scored **1** for **Point B1** and **0** for **Point B2**.

First Column	Second Column
Three correct values in part B (i) with values that are not expressed as decimal approximations	Correct end behavior statement in part B (ii) without use of limit notation
Three correct values in part B (i) with a decimal presentation error	Correct end behavior statement in part B (ii) with incorrect limit notation
Only one correct value in part B (i) with no incorrect values included (may have a decimal presentation error)	Correct limit statement in part B (ii) that is missing “ $x \rightarrow \infty$ ”
Only two correct values in part B (i) with no incorrect values included (may have a decimal presentation error)	

- C (i) Based on the table, which of the following function types best models function f : linear, quadratic, exponential, or logarithmic?
- (ii) Give a reason for your answer in part C (i) based on the relationship between the change in the output values of f and the change in the input values of f . Refer to the values in the table in your reasoning.

(i) An exponential function best models f .	Answer	Point C1
(ii) In this case, the input-value intervals all have length 1. Examining the ratios of successive output values gives $\frac{f(-1)}{f(-2)} = \frac{f(0)}{f(-1)} = \frac{f(1)}{f(0)} = \frac{f(2)}{f(1)} = 0.5$. Because the successive output values over equal-length input-value intervals are proportional, an exponential model is best.	Reasoning	Point C2

Scoring Notes for Part C

- **Point C1** is earned for a correct function model with no incorrect function models listed. A response of “exponential” earns **Point C1**.
- Both **Point C1** and **Point C2** may be earned in part C (ii) provided there is no incorrect response in part C (i).
- **Point C2** requires an implicit or explicit reference to output values and input values from the table as support for the reason. For example, $\frac{f(n+1)}{f(n)} = 0.5$ OR “successive output values are decreasing proportionately by a factor of 0.5.” The reasoning must demonstrate that the ratio of 0.5 applies to more than one pair of successive output values.
- A reason that references “exponential regression,” “ r values,” OR “ r^2 values” is not sufficient to earn **Point C2**.
- Special case: A response that indicates that f is best modeled by a **linear, quadratic, or logarithmic** function in part C (i) without a reason in part C (i) combined with a response in part C (ii) that provides both the correct answer and a correct reason is scored **0** for **Point C1** and **1** for **Point C2**.

Question 2: Modeling a Non-Periodic Context

Part A: Graphing calculator required

6 points

A musician released a new song on a streaming service. A streaming service is an online entertainment source that allows users to play music on their computers and mobile devices.

Several months later, the musician began using an app (at time $t = 0$) that counts the total number of plays for the song since its release. A “play” is a single stream of the song on the streaming service. The table gives the total number of plays, in thousands, for selected times t months after the musician began using the app.

At $t = 0$, the total number of plays was 25 thousand. At $t = 2$, the total number of plays was 30 thousand.

At $t = 4$, the total number of plays was 34 thousand.

Months after the musician began using the app	0	2	4
Total number of plays for the song since its release (thousands)	25	30	34

The total number of plays, in thousands, for the song since its release can be modeled by the function D given by $D(t) = at^2 + bt + c$, where $D(t)$ is the total number of plays, in thousands, for the song since its release, and t is the number of months after the musician began using the app.

Model Solution	Scoring
A (i) Use the given data to write three equations that can be used to find the values for constants a , b , and c in the expression for $D(t)$.	
(ii) Find the values for a , b , and c as decimal approximations.	
(i) Because $D(0) = 25$, $D(2) = 30$, and $D(4) = 34$, three equations to find a , b , and c are $a(0)^2 + b(0) + c = 25$ $a(2)^2 + b(2) + c = 30$ $a(4)^2 + b(4) + c = 34$	Three equations Point A1
(ii) $c = 25$ $4a + 2b = 5 \Rightarrow 16a + 8b = 20$ $16a + 4b = 9 \Rightarrow 16a + 4b = 9 \Rightarrow 4b = 11$ $b = \frac{11}{4} = 2.75$ $a = -0.125$ $D(t) = -0.125t^2 + 2.75t + 25$	Values of a , b , and c Point A2

General Scoring Notes for Question 2 Parts A, B, and C

- Decimal approximations must be accurate to three places after the decimal point by rounding or truncating. Decimal values of 0 in final digits need not be reported ($2.000 = 2.00 = 2.0 = 2$).
- A **decimal presentation error** occurs when a response is complete and correct, but the answer is reported to fewer digits than required.
- The first decimal presentation error in Question 2 does not earn the point. For each additional part of Question 2 that requires a decimal approximation and contains a decimal presentation error, the response is eligible to earn the point.
- Parts of Question 2 require decimal answers. The first response in Question 2 that is complete, correct, and uses exact values rather than decimal form does not earn the point. For each additional part of Question 2 that requires a decimal answer, a response that is complete, correct, and uses exact values rather than decimal form is eligible to earn the point.

Scoring Notes for Part A

- Point A1** is earned for presenting three equations involving a , b , and c that use the given input-output pairs.
- Point A2** is earned for correct values of a , b , and c **with or without** supporting work. If correct values are identified, work should be ignored.
- Point A2** is earned for correct values of a , b , and c presented as either standalone values OR in an expression for $D(t)$.
- A response is eligible to earn both **Point A1** and **Point A2** with a correct translation to “thousands.” Use of 25,000, 30,000, and 34,000 results in values of $a = -125$, $b = 2750$, and $c = 25,000$.

Partial Credit for Part A

A response that **does not** earn either **Point A1** or **Point A2** is eligible for **partial credit** in part A if the response has two correct equations in the presence of three equations involving a , b , and c AND one correct value.

Partial credit response is scored **1** for **Point A1** and **0** for **Point A2**.

- B**
- (i) Use the given data to find the average rate of change of the total number of plays for the song, in thousands per month, from $t = 0$ to $t = 4$ months. Express your answer as a decimal approximation. Show the computations that lead to your answer.
- (ii) Use the average rate of change found in part B (i) to estimate the total number of plays for the song, in thousands, for $t = 1.5$ months. Show the work that leads to your answer.

$$(i) \frac{D(4) - D(0)}{4 - 0} = \frac{(34 - 25)}{4} = 2.25$$

The average rate of change is 2.25 thousand plays per month.

Average rate of
change

Point B1

(ii) Estimates using the average rate of change can be given by

$$y = 25 + 2.25t.$$

For $t = 1.5$,

$$y = 25 + 2.25 \cdot 1.5 = 28.375.$$

The estimate of the total number of plays for the song for $t = 1.5$ months is 28.375 thousand.

Estimate using
average rate of
change

Point B2

Scoring Notes for Part B (i)–(ii)

- Point B1** and **Point B2** both require supporting work.
- Point B1** is earned for a correct decimal approximation in the presence of a quotient with a difference that uses the given data values. In part B (i) units are not needed and are ignored if presented.
- Another form of the secant line, derived from $D(4) = 34$, is $y = 34 + 2.25(t - 4)$.
- Eligibility for **Point B2**:
 - If a response in part B (i) has a decimal presentation error OR if a response in part B (i) is incorrect:
 - The reported value in part B (i) as the average rate of change can be used to arrive at an estimate in part B (ii). To earn **Point B2**, the estimate in part B (ii) must be consistent with both the reported value in part B (i) and the endpoint used in the supporting work in part B (ii).
- The final number in part B (ii) may be reported as 28 thousand provided the supporting work has a correct decimal approximation for the estimate.
- If a response has a correct translation to “thousands”:
 - The response is eligible to earn both **Point B1** and **Point B2**.
 - Use of 25,000 and 34,000 results in an answer of 2250 in part B (i).
 - If a response earned **Point B1** without a decimal presentation error, then an estimate of 28,375 earns **Point B2** in the presence of supporting work.
 - If a response in part B (i) has a decimal presentation error OR if a response in part B (i) is incorrect:
 - The reported value in part B (i) as the average rate of change can be used to arrive at an estimate in part B (ii). To earn **Point B2**, the estimate in part B (ii) must be consistent with both the reported value in part B (i) and the endpoint used in the supporting work in part B (ii).

Partial Credit for Part B (i)–(ii)

A response that **does not** earn either **Point B1** or **Point B2** is eligible for **partial credit** in part B if the response has one criteria from the first column AND one criteria from the second column.

Partial credit response is scored **1** for **Point B1** and **0** for **Point B2**.

First Column	Second Column
Correct quotient in part B (i) that uses the given data values that is not expressed as a decimal approximation	Correct estimate of 28.375 in part B (ii) that does not include supporting work
Correct quotient in part B (i) that uses the given data values and has a decimal presentation error	Correct or consistent supporting work in part B (ii) that does not provide an estimate
Correct average rate of change in part B (i) that does not include supporting work	

- B** (iii) Let A_t represent the estimate of the total number of plays for the song, in thousands, using the average rate of change found in part B (i). For $A_{1.5}$ found in part B (ii), it can be shown that $A_{1.5} < D(1.5)$.

Explain why, in general, $A_t < D(t)$ for all t , where $0 < t < 4$. Your explanation should include a reference to the graph of D and its relationship to A_t .

(iii) The estimate A_t is the y -coordinate of a point on the secant line that passes through $(0, D(0))$ and $(4, D(4))$. Because the graph of D is concave down on the interval $0 < t < 4$, this secant line is below the graph of D on the interval $0 < t < 4$. Therefore, the estimate using the average rate of change A_t is less than the value of $D(t)$ for all t on the interval $0 < t < 4$.

Explanation

Point B3**Scoring Notes for Part B (iii)**

To earn **Point B3**, the explanation must include:

- The graph of D is concave down OR the rate of change of D is decreasing.
- A reference to the use of a secant line on $0 < t < 4$ OR the use of a linear function with reference to t -values 0 and 4 that provide the placement of the line.

- C** The quadratic function model D has exactly one absolute minimum or one absolute maximum. That minimum or maximum can be used to determine a domain restriction for D .

Based on the context of the problem, explain how that minimum or maximum can be used to determine a boundary for the domain of D .

The total number of plays for the song must be nonnegative and never decreasing. The quadratic function D has an absolute maximum at $t = 11$. The function D increases before $t = 11$ and decreases after $t = 11$. Because D is also positive for positive t -values less than $t = 11$, the domain of model D is an interval with right endpoint $t = 11$.

Explanation

Point C1**Scoring Notes for Part C**

- A response that focuses on the left endpoint of the domain is eligible for **Point C1**.
- To earn **Point C1**, the explanation must include one of the following:
 - The total number of plays cannot decrease AND the function model D decreases after the time it achieves its absolute maximum.
 - The location of the absolute maximum at $t = 11$ is not required but must be correct if included.
 - The total number of plays is nonnegative AND the left endpoint of the domain is the only value of t at which $D(t)$ changes from negative to positive.
 - $D(t)$ changes from negative to positive at $t = -6.916$. This value is not required, but if included it must be accurate to at least one place after the decimal point, rounded or truncated.

Question 3: Modeling a Periodic Context**Part B: Graphing calculator not allowed****6 points**

For a guitar to make a sound, the strings need to vibrate, or move up and down or back and forth, in a motion that can be modeled by a periodic function.

At time $t = 0$ seconds, point X on one vibrating guitar string starts at its highest position, 2 millimeters above its resting position. Then it passes through its resting position and moves to its lowest position, 2 millimeters below the resting position. Point X then passes through its resting position and returns to 2 millimeters above the resting position. This motion occurs 200 times in 1 second.

The sinusoidal function h models how far point X is from its resting position, in millimeters, as a function of time t , in seconds. A positive value of $h(t)$ indicates the point is above the resting position; a negative value of $h(t)$ indicates the point is below the resting position.

Model Solution	Scoring
A The graph of h and its dashed midline for two full cycles is shown. Five points, F, G, J, K, and P, are labeled on the graph. No scale is indicated, and no axes are presented. Determine possible coordinates $(t, h(t))$ for the five points: F, G, J, K, and P.	
	$h(t)$-coordinates Point A1
Note: t-coordinates will vary. A correct set of coordinates for one full cycle of h as pictured is acceptable.	t-coordinates Point A2

Scoring Notes for Part A

- No supporting work is required.
- $h(t)$ -coordinates and/or t -coordinates may appear in a list.
- Negative t -coordinates are acceptable. Fractions do not need to be reduced; equivalent fractions and exact decimal values are acceptable.
- t -coordinates must be $0 + \frac{4}{800}k$, $\frac{1}{800} + \frac{4}{800}k$, $\frac{2}{800} + \frac{4}{800}k$, $\frac{3}{800} + \frac{4}{800}k$, $\frac{4}{800} + \frac{4}{800}k$ for a specific integer k .

- If the graph is used to record coordinates, that work is scored. In this case, other work is considered scratchwork and is not scored. Use of the graph is not required.

Partial Credit for Part A

A response that **does not** earn either **Point A1** or **Point A2** is eligible for **partial credit** in part A if the response meets one of the following criteria:

- All 5 points are in the form $(h(t), t)$ with correct input values and correct output values swapped.
- 3 out of the 5 points are correct.
- All 5 points $(t, h(t))$ meet these requirements:
 - t -coordinates are in arithmetic sequence with $\Delta t = \frac{1}{800}$.
 - $h(t)$ -coordinates are such that
 - F and P have **same** $h(t)$ -coordinate.
 - G and K have **same** $h(t)$ -coordinate, which is **less than** $h(t)$ -coordinate of F and P .
 - Difference in $h(t)$ -coordinates for F and G **equals** Difference in $h(t)$ -coordinates for G and J .

Partial credit response is scored **0** for **Point A1** and **1** for **Point A2**.

- B** The function h can be written in the form $h(t) = a \sin(b(t + c)) + d$. Find values of constants a , b , c , and d .

$$\frac{2\pi}{b} = \frac{1}{200}, \text{ so } b = 400\pi$$

$$d = 0$$

Method 1:

Using $a = 2$,

$$c = \frac{1}{800} \text{ OR } c = \frac{1}{800} + \frac{4}{800}k, \text{ for any integer } k$$

$a =$	<u>2</u>
$b =$	<u>400π</u>
$c =$	<u>$\frac{1}{800}$</u>
$d =$	<u>0</u>

For example, $h(t) = 2 \sin\left(400\pi\left(t + \frac{1}{800}\right)\right)$. Based on horizontal shifts, there are other correct forms for $h(t)$.

Vertical transformations:
Values of a and d

Point B1

Horizontal transformations:
Values of b and c

Point B2

Method 2:Using $a = -2$,

$$c = -\frac{1}{800} \text{ OR } c = -\frac{1}{800} + \frac{4}{800}k, \text{ for any integer } k$$

$a =$	-2
$b =$	400π
$c =$	$-\frac{1}{800}$
$d =$	0

For example, $h(t) = -2\sin\left(400\pi\left(t - \frac{1}{800}\right)\right)$. Based onhorizontal shifts, there are other correct forms for $h(t)$.**Scoring Notes for Part B**

- No supporting work is required.
- Points are earned for correct values in a list OR for correct values in an expression for $h(t)$. Only one of these answer presentations is required.
- Fractions do not need to be reduced; equivalent fractions and exact decimal values are acceptable.
- If the answer box is used to record values, that work is scored. In this case, other work is considered scratchwork and is not scored. Use of the answer box is not required.
- Point B1** and **Point B2** may be earned based on the correct use of an imported response from part A that meets these criteria:
 - $a \neq 1$, $b \neq 1$, and $c \neq 0$.
 - All 5 points $(t, h(t))$ from part A meet these requirements:
 - t -coordinates are in arithmetic sequence with $\Delta t = \frac{1}{800}$.
 - $h(t)$ -coordinates are such that
 - F and P have **same** $h(t)$ -coordinate.
 - G and K have **same** $h(t)$ -coordinate, which is **less than** $h(t)$ -coordinate of F and P .
 - Difference in $h(t)$ -coordinates for F and G **equals** Difference in $h(t)$ -coordinates for G and J .

Partial Credit for Part BA response that **does not** earn either **Point B1** or **Point B2** is eligible for **partial credit** in part B if the response meets one of the following criteria:

- Correct values of a and b [Values of a and b could be $\pm$]
- Correct values of b and d [Value of b could be $\pm$]
- Response uses $h(t) = a\cos(b(t+c)) + d$ with values as follows:
 - $a = 2$; $b = 400\pi$; $c = 0 + \frac{4}{800}k$, for a specific integer k ; $d = 0$
 - $a = -2$; $b = 400\pi$; $c = -\frac{2}{800} + \frac{4}{800}k$, for a specific integer k ; $d = 0$

Partial credit response is scored **1** for **Point B1** and **0** for **Point B2**.**C** Refer to the graph of h in part A. The t -coordinate of G is t_1 , and the t -coordinate of J is t_2 .

- (i) On the interval (t_1, t_2) , which of the following is true about h ?
- h is positive and increasing.
 - h is positive and decreasing.
 - h is negative and increasing.
 - h is negative and decreasing.
- (ii) On the interval (t_1, t_2) , describe the concavity of the graph of h and determine whether the rate of change of h is increasing or decreasing.

(i) Choice d.	Function behavior	Point C1
(ii) The graph of h is concave up on the interval (t_1, t_2) , and the rate of change of h is increasing on the interval (t_1, t_2) .	Concavity of graph and behavior of rate of change	Point C2

Scoring Notes for Part C

- No supporting work is required.
- Point C1** is earned only for a correct answer of “d” OR “negative and decreasing.” If both the letter choice and written description are included, the written description is scored.
- To earn **Point C2**, both descriptions must be correct. **Point C2** is not earned for a response that only includes “the graph of h is concave up” OR only includes “the rate of change of h is increasing.”
- To earn **Point C2**, “concave up” AND “increasing” is acceptable.
- To earn **Point C2**, “concave up” AND “function h is decreasing at an increasing rate” is acceptable.
- A response with an isolated statement “decreasing at an increasing rate” does not earn **Point C2**. The implied subject is “the rate of change of h .”
- A response with a statement that “the rate of change of h is increasing at an increasing (or decreasing) rate” does not earn **Point C2**. Analysis to make such a conclusion requires calculus.
- Point C2** cannot be earned if there are any errors in part C (ii).

Question 4: Symbolic Manipulations
Part B: Graphing calculator not allowed
6 points**Directions:**

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

	Model Solution	Scoring
A	The functions g and h are given by $g(x) = 2\log_3 x$ $h(x) = 4\cos^2 x$ (i) Solve $g(x) = 4$ for values of x in the domain of g. (ii) Solve $h(x) = 3$ for values of x in the interval $\left[0, \frac{\pi}{2}\right)$.	
	(i) $g(x) = 4$ $2\log_3 x = 4$ $\log_3 x = 2$ $3^2 = x$ $x = 9$	Solution to $g(x) = 4$ Point A1
	(ii) $h(x) = 3$ $4\cos^2 x = 3$ $\cos^2 x = \frac{3}{4}$ $\cos x = \pm \frac{\sqrt{3}}{2}$ Because x is in $\left[0, \frac{\pi}{2}\right)$, $x = \frac{\pi}{6}$	Solution to $h(x) = 3$ Point A2

Scoring Notes for Part A

- Point A1** and **Point A2** both require supporting work. “Scratchwork” can be ignored; the use of a variable other than x is acceptable. Arithmetic errors following a complete and correct solution may be considered scratchwork. The use of “ $x =$ ” is not required.
- A logarithmic expression that adds one or both parentheses around the full argument of the logarithm is eligible to earn **Point A1**.
- A response that includes correct values of x outside of the interval $\left[0, \frac{\pi}{2}\right)$ is eligible to earn

Point A2 (e.g., $x = \frac{5\pi}{6}$, $x = \frac{7\pi}{6}$, or $x = \frac{11\pi}{6}$).

- The use of $\pm$ is not required in supporting work for **Point A2**.
- Where applicable, answers that have not been evaluated according to bullets two and three in the Directions do not earn the point. Rationalizing denominators is not required.

Partial Credit for Part A

A response that **does not** earn either **Point A1** or **Point A2** is eligible for **partial credit** in part A if the response has one criteria from the first column AND one criteria from the second column.

Partial credit response is scored **1** for **Point A1** and **0** for **Point A2**.

First Column	Second Column
Correct answer in part A (i) without supporting work.	Correct answer in part A (ii) without supporting work.
Correct answer in part A (i) with supporting work, but the answer has not been evaluated according to bullets two and three in the Directions (e.g., $x = 3^2$). No incorrect work.	Correct answer in part A (ii) with supporting work, but the answer has not been evaluated according to bullets two and three in the Directions. This includes an answer of $x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$. No incorrect work.
Answer in part A (i) is reported as $x^2 = 3^4$ OR $x^2 = 81$. No incorrect work follows.	Answer in part A (ii) is reported as $\cos x = \pm \frac{\sqrt{3}}{2}$ OR $\cos x = \frac{\sqrt{3}}{2}$. No incorrect work follows.

B The functions j and k are given by

$$j(x) = \log_2 x + 3\log_2 2$$

$$k(x) = \frac{6}{\tan x (\csc^2 x - 1)}$$

(i) Rewrite $j(x)$ as a single logarithm base 2 without negative exponents in any part of the expression.

Your result should be of the form $\log_2(\text{expression})$.

(ii) Rewrite $k(x)$ as an expression in which $\tan x$ appears exactly once and no other trigonometric functions are involved.

$$(i) j(x) = \log_2 x + 3\log_2 2$$

$$j(x) = \log_2 x + \log_2 2^3$$

$$j(x) = \log_2(8x), x > 0$$

Expression for $j(x)$ **Point B1**

$$(ii) k(x) = \frac{6}{\tan x (\csc^2 x - 1)}$$

$$k(x) = \frac{6}{\tan x (\cot^2 x)}$$

$$k(x) = \frac{6}{\cot x}$$

$$k(x) = 6 \tan x, \tan x \neq 0, \cot x \neq 0$$

Expression for $k(x)$ **Point B2**

Scoring Notes for Part B

- **Point B1** is earned with a correct expression for $j(x)$ without supporting work, provided no incorrect work is included. “Scratchwork” can be ignored; the use of a variable other than x is acceptable. The use of “ $j(x) =$ ” is not required.
- **Point B2** requires supporting work. Scratchwork can be ignored; the use of a variable other than x is acceptable. The use of “ $k(x) =$ ” is not required.
- Domain restrictions are not required to be included and are not scored regardless if correct or incorrect.
- Where applicable, answers that have not been evaluated according to bullets two and three in the Directions do not earn the point.
- A logarithmic expression that is missing one or both parentheses around the full argument of the logarithm is still eligible to earn **Point B1**.
- If a response is presented as a complex fraction, the complex fraction must be unambiguous in structure. Parentheses must be used correctly, and/or the fraction bars must be clearly and correctly proportioned.

Partial Credit for Part B

A response that **does not** earn either **Point B1** or **Point B2** is eligible for **partial credit** in part B if the response has one criteria from the first column AND one criteria from the second column.

Partial credit response is scored **1** for **Point B1** and **0** for **Point B2**.

First Column	Second Column
Expression in part B (i) is reported as $\log_2 x + 3$. No incorrect work follows.	Correct expression in part B (ii) without supporting work.
Expression in part B (i) is reported as $\log_2(2^3 \cdot x)$. No incorrect work follows.	Expression in part B (ii) is reported as $\frac{6}{\cot x}$ OR $\frac{6 \sin x}{\cos x}$. No incorrect work follows.
Expression in part B (i) is reported using logarithm base b , $b > 0$, and $b \neq 2$, and has the correct argument.	Expression in part B (ii) includes a correct application of a Pythagorean identity with no incorrect work.

C The function m is given by

$$m(x) = e^{2x} - e^x - 12.$$

Find all input values in the domain of m that yield an output value of 0.

$$m(x) = 0 \Rightarrow e^{2x} - e^x - 12 = 0$$

$$(e^x)^2 - e^x - 12 = 0$$

$$\text{Let } y = e^x.$$

$$y^2 - y - 12 = 0$$

$$(y - 4)(y + 3) = 0$$

$$y - 4 = 0 \text{ or } y + 3 = 0$$

$$e^x = 4 \text{ or } e^x = -3 \Rightarrow x = \ln 4$$

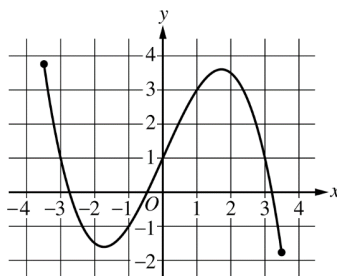
Quadratic form with e^x **Point C1**

Value of x **Point C2**

Scoring Notes for Part C

- **Point C1** and **Point C2** both require supporting work. “Scratchwork” can be ignored; the use of a variable other than x is acceptable. The use of “ $x =$ ” is not required.
- **Point C1** is earned for a substitution of $y = e^x$ and factored form of $(y - 4)(y + 3)$ [the use of a variable other than y is acceptable] OR for presenting $m(x)$ in factored form as $(e^x - 4)(e^x + 3)$.
- To earn **Point C2**, no incorrect values for x are included.

Question 1: Function Concepts
Part A: Graphing calculator required

6 pointsGraph of f

The figure shows the graph of the function f on its domain of $-3.5 \leq x \leq 3.5$. The points $(-3, 1)$, $(0, 1)$, and $(3, 1)$ are on the graph of f . The function g is given by $g(x) = 2.916 \cdot (0.7)^x$.

Model Solution**Scoring**

- (A) (i) The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(3)$ as a decimal approximation, or indicate that it is not defined.
(ii) Find all values of x for which $f(x) = 1$, or indicate that there are no such values.

(i) $h(3) = g(f(3)) = g(1) = 2.041$	Value	1 point
(ii) From the graph, $f(x) = 1$ when $x = -3$, $x = 0$, and $x = 3$.	Values	1 point

General Scoring Notes for Question 1 Parts (A), (B), and (C):

- Decimal approximations must be correct to three places after the decimal point by rounding or truncating. Decimal values of 0 in final digits need not be reported ($2.000 = 2.00 = 2.0 = 2$).
- A **decimal presentation error** occurs when a response is complete and correct, but the answer is reported to fewer digits than required.
- The first decimal presentation error in Question 1 does not earn the point. For each additional part of Question 1 that requires a decimal approximation and contains a decimal presentation error, the response is eligible to earn the point.

Scoring notes:

- The first point is earned for a correct decimal approximation of 2.041.
- The second point does not require supporting work.

- A response that does not earn either point in Part (A) is eligible for **partial credit** in Part (A) if the response has one criteria from the first column AND one criteria from the second column. Partial credit response is scored **0-1** in Part (A).

First Column	Second Column
Correct value in (i) that is not expressed as a decimal approximation	Only one correct value in (ii) with no incorrect values included
Correct value in (i) with a decimal presentation error	Only two correct values in (ii) with no incorrect values included

- (B) (i) Find all values of x , as decimal approximations, for which $g(x) = 2$, or indicate that there are no such values.
(ii) Determine the end behavior of g as x increases without bound. Express your answer using the mathematical notation of a limit.

(i) $g(x) = 2 \Rightarrow 2.916(0.7)^x = 2$ $x = 1.057$	Value	1 point
(ii) As x increases without bound, the output values of g get arbitrarily close to 0. Therefore, $\lim_{x \rightarrow \infty} g(x) = 0$.	End behavior with limit notation	1 point

Scoring notes:

- The first point is earned for a correct decimal approximation of 1.057. No incorrect values may be included.
- The second point requires a correct limit statement with four components: "lim," " $x \rightarrow \infty$," the function g , and 0. Examples that earn the point include:
 - $\lim_{x \rightarrow \infty} g(x) = 0$ OR $\lim_{x \rightarrow \infty} g = 0$
 - $\lim_{x \rightarrow \infty} g(x) \rightarrow 0$ OR $\lim_{x \rightarrow \infty} g \rightarrow 0$
 - $\lim_{x \rightarrow \infty} g(x) \ 0$ OR $\lim_{x \rightarrow \infty} g \ 0$

If the response includes an additional, complete limit statement (e.g., $\lim_{x \rightarrow -\infty} g(x) = \infty$), the value of the limit must be correct.

- A response that does not earn either point in Part (B) is eligible for **partial credit** in Part (B) if the response has one criteria from the first column AND one criteria from the second column. Partial credit response is scored **1-0** in Part (B).

First Column	Second Column
Correct answer in (i) that is not expressed as a decimal approximation	Correct end behavior statement in (ii) without use of limit notation
Correct value in (i) with a decimal presentation error	Correct end behavior statement in (ii) with incorrect limit notation
	Correct limit statement in (ii) that is missing " $x \rightarrow \infty$ "

- (C) (i) Determine if f has an inverse function.
(ii) Give a reason for your answer based on the definition of a function and the graph of $y = f(x)$.

(i) f does not have an inverse function on its domain of $-3.5 \leq x \leq 3.5$.	Answer	1 point
(ii) There are output values of f that are not mapped from unique input values; for example, $f(-3) = f(0) = f(3) = 1$.	Reason	1 point

Scoring notes:

- The first point is earned for a correct answer.
- Both points may be earned in (ii) provided there is no incorrect response in (i).
- The second point requires an implicit or explicit reference to the definition of a function AND support for the reason by referencing specific function values.
- A response such as " f does not have an inverse function because $f(-3) = f(0) = 1$ " OR " f does not have an inverse function because there are two input values mapped to 1" earns both points.
- A response such as " f is not one-to-one" OR " f fails the horizontal line test" OR "There are output values that are not mapped from unique input values" is not sufficient to earn the second point.
- The second point cannot be earned if there are any errors in Part (C) (ii).
- A response that indicates that f **has** an inverse function in Part (C) (i) without a reason in Part (C) (i) **combined** with a response in Part (C) (ii) that provides both the correct answer **and** a correct reason is scored **0-1**.

Total for question 1 6 points

Question 3: Modeling a Periodic Context
Part B: Graphing calculator not allowed

6 points



Note: Figure not drawn to scale.

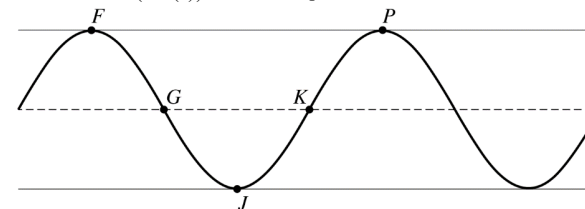
The tire of a car has a radius of 9 inches, and a person rolls the tire forward at a constant rate on level ground, as shown in the figure. Point W on the edge of the tire touches the ground at time $t = \frac{1}{2}$ second. The tire completes a full rotation, and the next time W touches the ground is at time $t = \frac{5}{2}$ seconds. The maximum height of W above the ground is 18 inches. As the tire rolls, the height of W above the ground periodically increases and decreases.

The sinusoidal function h models the height of point W above the ground, in inches, as a function of time t , in seconds.

Model Solution

Scoring

- (A) The graph of h and its dashed midline for two full cycles is shown. Five points, F , G , J , K , and P , are labeled on the graph. No scale is indicated, and no axes are presented. Determine possible coordinates $(t, h(t))$ for the five points: F , G , J , K , and P .



F has coordinates $\left(\frac{3}{2}, 18\right)$.	$h(t)$ -coordinates	1 point
G has coordinates $(2, 9)$.	t -coordinates	1 point
J has coordinates $\left(\frac{5}{2}, 0\right)$.		
K has coordinates $(3, 9)$.		
P has coordinates $\left(\frac{7}{2}, 18\right)$.		
OR		

F has coordinates $\left(-\frac{1}{2}, 18\right)$.

G has coordinates $(0, 9)$.

J has coordinates $\left(\frac{1}{2}, 0\right)$.

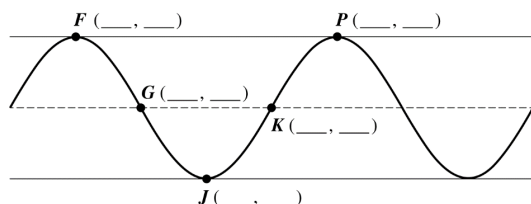
K has coordinates $(1, 9)$.

P has coordinates $\left(\frac{3}{2}, 18\right)$.

Note: t -coordinates will vary. A correct set of coordinates for one full cycle of h as pictured is acceptable.

Scoring notes:

- No supporting work is required.
- $h(t)$ -coordinates and/or t -coordinates may appear in a list. Negative t -coordinates are acceptable.
- t -coordinates must be $-\frac{1}{2} + 2k, 0 + 2k, \frac{1}{2} + 2k, 1 + 2k, \frac{3}{2} + 2k$, for a specific integer k .
- If the graph is used to record coordinates, that work is scored. In this case, other work is considered scratchwork and is not scored. Use of the graph is not required.



- A response that does not earn either point in Part (A) is eligible for **partial credit** in Part (A) if the response meets one of the following criteria. Partial credit response is scored **0-1** in Part (A).
 - All 5 points in the form $(h(t), t)$ with correct input values and correct output values swapped
 - 3 correct points out of the 5 points
 - All 5 points $(t, h(t))$ meet these requirements:
 - t -coordinates in arithmetic sequence with $\Delta t = \frac{1}{2}$
 - $h(t)$ -coordinates are such that
 - F and P have same $h(t)$ -coordinate
 - G and K have same $h(t)$ -coordinate, which is less than $h(t)$ -coordinate of F and P
 - Difference in $h(t)$ -coordinates for F and G equals difference in $h(t)$ -coordinates for G and J

- (B) The function h can be written in the form $h(t) = a \sin(b(t + c)) + d$. Find values of constants a , b , c , and d .

$$h(t) = a \sin(b(t + c)) + d$$

$$a = 9$$

$$\frac{2\pi}{b} = 2, \text{ so } b = \frac{2\pi}{2} = \pi$$

$$c = -1$$

$$d = 9$$

$$h(t) = 9 \sin(\pi(t - 1)) + 9$$

OR

$$a = -9$$

$$\frac{2\pi}{b} = 2, \text{ so } b = \frac{2\pi}{2} = \pi$$

$$c = 0$$

$$d = 9$$

$$h(t) = -9 \sin(\pi t) + 9$$

Note: Based on horizontal shifts and reflections, there are other correct forms for $h(t)$.

Vertical transformations:
Values of a and d

1 point

Horizontal transformations:
Values of b and c

1 point

Scoring notes:

- No supporting work is required.
- Points are earned for correct values in a list OR for correct values in an expression for $h(t)$. Only one of these answer presentations is required.
- If the answer box is used to record values, that work is scored. In this case, other work is considered scratchwork and is not scored. Use of the answer box is not required.

$a =$ _____
 $b =$ _____
 $c =$ _____
 $d =$ _____

- Other correct values of c :
 - $h(t) = 9 \sin(\pi(t + c)) + 9 \Rightarrow c = -1 + 2k$, for any integer k
 - $h(t) = -9 \sin(\pi(t + c)) + 9 \Rightarrow c = 0 + 2k$, for any integer k
- Full credit for Part (B) is possible based on the correct use of an imported response from Part (A) that meets these criteria:
 - $a \neq 1$, $b \neq 1$, $d \neq 0$, and if $a > 0$, then $c \neq 0$
 - All 5 points $(t, h(t))$ meet these requirements:
 - t -coordinates in arithmetic sequence with $\Delta t = \frac{1}{2}$
 - $h(t)$ -coordinates are such that
 - F and P have same $h(t)$ -coordinate
 - G and K have same $h(t)$ -coordinate, which is less than $h(t)$ -coordinate of F and P
 - Difference in $h(t)$ -coordinates for F and G equals difference in $h(t)$ -coordinates for G and J

- A response that does not earn either point in Part (B) is eligible for **partial credit** in Part (B) if the response meets one of the following criteria. Partial credit response is scored **1-0** in Part (B).
 - Values of a and b [Values of a and b could be $\pm$]
 - Values of b and d [Value of b could be $\pm$]
 - Response uses $h(t) = a\cos(b(t+c)) + d$ with values as follows:
 - $a = 9$; $b = \pi$; $c = -\frac{3}{2} + 2k$, for a specific integer k ; $d = 9$
 - $a = -9$; $b = \pi$; $c = -\frac{1}{2} + 2k$, for a specific integer k ; $d = 9$

(C) Refer to the graph of h in part (A). The t -coordinate of K is t_1 , and the t -coordinate of P is t_2 .

- (i) On the interval (t_1, t_2) , which of the following is true about h ?
- h is positive and increasing.
 - h is positive and decreasing.
 - h is negative and increasing.
 - h is negative and decreasing.
- (ii) Describe how the rate of change of h is changing on the interval (t_1, t_2) .

(i) Choice a.	Function behavior	1 point
(ii) Because the graph of h is concave down on the interval (t_1, t_2) , the rate of change of h is decreasing on the interval (t_1, t_2) .	Change in rate of change	1 point

Scoring notes:

- No supporting work is required.
- The first point is earned for a correct answer of “a” OR “positive and increasing.” If both the letter choice and written description are included, the written description is scored.
- To earn the second point, “decreasing” OR “function h is increasing at a decreasing rate” is acceptable. If concavity of the graph of h is referenced, it must be correct.
- The second point is not earned for a response that only includes “the graph of h is concave down.”
- A response with a statement that the rate of change of h is decreasing at an increasing (or decreasing) rate does not earn the second point. Analysis to make such a conclusion requires calculus.
- The second point is not earned for a response that states “increasing at a decreasing rate” without a subject. The implied subject is “the rate of change of h .”
- The second point cannot be earned if there are any errors in Part (C) (ii).

Total for question 3 6 points