Week 17

AP Precalculus

Homework bundle for this week

本周作业合集

exercises.lol

Contents 目录

3. Trigonometric and Polar Functions

Starter Quiz · 课前小测

AP Precalculus Name: ____________ Score: ______

1. A **periodic** function is one that…
   - ☐ repeats the same pattern at regular intervals
   - ☐ always increases
   - ☐ never repeats
   - ☐ is a straight line
2. In a right triangle, $\sin\theta$ equals…
   - ☐ opposite ÷ hypotenuse
   - ☐ adjacent ÷ hypotenuse
   - ☐ opposite ÷ adjacent
   - ☐ hypotenuse ÷ opposite
3. What is $\sin 0$?

   ____
4. The parent sine graph $y = \sin x$ starts (at $x = 0$) at height…
   - ☐ 0, then rises
   - ☐ 1, then falls
   - ☐ -1, then rises
   - ☐ 2, then falls
5. In $y = a\sin(b(x-c)) + d$, the **amplitude** is…
   - ☐ $|a|$
   - ☐ b
   - ☐ d
   - ☐ c
6. Increasing $|a|$ in $y = a\sin(bx) + d$ makes the wave…
   - ☐ taller (bigger amplitude)
   - ☐ faster (shorter period)
   - ☐ shift sideways
   - ☐ lift its midline

7. The horizontal length of one complete cycle is called the ______.

   ____
8. What is $\cos 0$?

   ____
9. In $y = a\sin(b(x-c)) + d$, the midline is the line $y =$ ______.

   ____
10. The midline of a fitted sinusoid is the ______ of the highest and lowest data values.

    ____

AP Precalculus

1. **repeats the same pattern at regular intervals**

   A **periodic** function cycles through the same values again and again — like tides or a spinning wheel.

2. **opposite ÷ hypotenuse**

   SOH: **sine** is opposite over hypotenuse. CAH: cosine is adjacent over hypotenuse. TOA: tangent is opposite over adjacent.

3. **0**

   At angle 0, the unit-circle point is $(1, 0)$, so the y-coordinate $\sin 0 = 0$.

4. 0**, then rises**

   Since $\sin 0 = 0$, the sine wave begins at the midline and rises to its peak of 1.

5. $|a|$

   The coefficient a stretches the wave vertically, so the **amplitude** is $|a|$.

6. **taller (bigger amplitude)**

   $|a|$ is the **amplitude**, so a larger $|a|$ stretches the wave taller.

7. **period**

   The **period** is how far along the input axis you go before the pattern repeats.

8. **1**

   At angle 0, the point is $(1, 0)$, so the x-coordinate $\cos 0 = 1$.

9. **d**

   Adding d shifts the whole wave up, so the **midline** sits at $y = d$.

10. **average**

    Midline $= \dfrac{\max + \min}{2}$; the **amplitude** is half the distance between them.

AP PRECALCULUS • EXERCISE SHEET

3.1 Periodic Phenomena

Name: ____________ Class: _________ Date: __________ **Total: 9 marks**

KEY WORDS period 周期 • amplitude 振幅 • midline 中线 • periodic 周期性 • cycle 周期段

Objective

Build the skills to answer exam questions on **periodic phenomena**.

You must be able to:

- identify the **period 周期**, **amplitude 振幅**, and **midline 中线** of a periodic function

1 Worked examples

Study these first. Each one shows the method for a question type used later.

Periodic functions

A **periodic** function repeats every **period** P (the length of one cycle). Its:

- **amplitude** $= \dfrac{\max - \min}{2}$;
- **midline** $= \dfrac{\max + \min}{2}$ (the horizontal centre).

Example

Max 8, min 2: amplitude $= 3$, midline $= 5$.

2 Practice

2.1 Define the period of a periodic function. [1]

2.2 State how the amplitude relates to the max and min. [1]

2.3 A wave has max 8 and min 2. Find the amplitude and midline. [2]

3 Exam-style questions

3.1 The period of a periodic function is [1]

- **A** its height
- **B** the length of one cycle
- **C** its maximum
- **D** its slope

3.2 The amplitude equals [1]

- **A** $\max - \min$
- **B** $\dfrac{\max - \min}{2}$
- **C** $\dfrac{\max + \min}{2}$
- **D** $\max$

3.3 A tide has maximum height 6 m and minimum 0 m, repeating every 12 h.

(a) Find the amplitude. [1]

(b) Find the midline. [1]

(c) State the period. [1]

Solutions

2.1 the length of one complete cycle.

2.2 amplitude $= \dfrac{\max - \min}{2}$.

2.3 amplitude $= \dfrac{8-2}{2} = 3$; midline $= \dfrac{8+2}{2} = 5$.

3.1 **B**.

3.2 **B**.

3.3 (a) $\dfrac{6-0}{2} = 3$ m. (b) $\dfrac{6+0}{2} = 3$ m. (c) 12 h.

AP PRECALCULUS • EXERCISE SHEET

3.2 Sine, Cosine, and Tangent

Name: ____________ Class: ________ Date: __________ **Total: 9 marks**

Objective

Build the skills to answer exam questions on **sine, cosine, and tangent**.

You must be able to:

- use the right-triangle ratios **SOH-CAH-TOA**
- relate $\tan\theta$ to $\sin\theta$ and $\cos\theta$

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ **Right-triangle ratios**

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}, \quad \cos\theta = \frac{\text{adj}}{\text{hyp}}, \quad \tan\theta = \frac{\text{opp}}{\text{adj}}.$$

Also $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$.

■ **Example**

A 3-4-5 right triangle (opp 3, adj 4, hyp 5): $\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$, $\tan\theta = \frac{3}{4}$.

2 Practice

2.1 State $\sin\theta$ in terms of the triangle sides. [1]

2.2 State $\tan\theta$ as a ratio of sin and cos. [1]

2.3 A right triangle has opp 3, adj 4, hyp 5. Find $\sin\theta$ and $\cos\theta$. [2]

3 Exam-style questions

3.1 $\cos\theta$ equals [1]

- **A** opp/hyp
- **B** adj/hyp
- **C** opp/adj
- **D** hyp/adj

3.2 $\tan\theta$ equals [1]

- **A** $\sin\theta \cdot \cos\theta$
- **B** $\sin\theta / \cos\theta$
- **C** $\cos\theta / \sin\theta$
- **D** $1/\sin\theta$

3.3 A right triangle has opp 5, adj 12, hyp 13.

(a) Find $\sin\theta$. [1]

(b) Find $\cos\theta$. [1]

(c) Find $\tan\theta$. [1]

Solutions

2.1 $\sin\theta = \text{opp}/\text{hyp}$.

2.2 $\tan\theta = \sin\theta/\cos\theta$.

2.3 $\sin\theta = \frac{3}{5}$; $\cos\theta = \frac{4}{5}$.

3.1 **B**.

3.2 **B**.

3.3 (a) $\frac{5}{13}$. (b) $\frac{12}{13}$. (c) $\frac{5}{12}$.

AP PRECALCULUS • EXERCISE SHEET

3.3 Sine and Cosine Function Values

Name: ________ Class: ________ Date: ________ **Total: 9 marks**

KEY WORDS unit circle 单位圆 • radians 弧度

Objective

Build the skills to answer exam questions on **sine and cosine function values**.

You must be able to:

- use the **unit circle 单位圆**: the point at angle θ is $(\cos\theta, \sin\theta)$
- convert between degrees and **radians 弧度**

1 Worked examples

Study these first. Each one shows the method for a question type used later.

The unit circle

On the unit circle, the point at angle θ is $(\cos\theta, \sin\theta)$.

Radians: $\pi = 180°$.

Key values: $\cos 0 = 1$, $\sin 0 = 0$; $\cos\frac{\pi}{2} = 0$, $\sin\frac{\pi}{2} = 1$.

Using the unit circle

Find $\cos\frac{\pi}{3}$ and $\sin\frac{\pi}{3}$. The angle $\frac{\pi}{3} = 60°$ gives the point $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, so $\cos\frac{\pi}{3} = \frac{1}{2}$ and $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$. The x-coordinate is the cosine; the y-coordinate is the sine.

2 Practice

2.1 State the coordinates of the unit-circle point at angle θ. [1]

2.2 Convert 180° to radians. [1]

2.3 State $\sin\frac{\pi}{2}$ and $\cos\frac{\pi}{2}$. [2]

3 Exam-style questions

3.1 On the unit circle, the point at angle θ is [1]

- **A** $(\sin\theta, \cos\theta)$
- **B** $(\cos\theta, \sin\theta)$
- **C** $(\tan\theta, 1)$
- **D** $(1, \theta)$

3.2 π radians equals [1]

- **A** 90°
- **B** 180°
- **C** 270°
- **D** 360°

3.3 Evaluate each.

(a) $\cos 0$. [1]

(b) $\sin 0$. [1]

(c) $\sin\frac{\pi}{2}$. [1]

Solutions

2.1 $(\cos\theta, \sin\theta)$.

2.2 π radians.

2.3 $\sin\frac{\pi}{2} = 1$; $\cos\frac{\pi}{2} = 0$.

3.1 **B**.

3.2 **B**.

3.3 (a) 1. (b) 0. (c) 1.

AP PRECALCULUS • EXERCISE SHEET

3.4 Sine and Cosine Function Graphs

Name: ____________ Class: ________ Date: __________ **Total: 9 marks**

KEY WORDS period 周期 • midline 中线 • amplitude 振幅

Objective

Build the skills to answer exam questions on **sine and cosine function graphs**.

You must be able to:

- state the period, amplitude, midline, and range of $y = \sin x$ and $y = \cos x$
- relate $\cos x$ to a shifted $\sin x$

1 Worked examples

Study these first. Each one shows the method for a question type used later.

The basic graphs

$y = \sin x$ and $y = \cos x$ each have:

- period 2π, amplitude 1, midline $y = 0$, range $[-1, 1]$.

They are shifts of each other: $\cos x = \sin\left(x + \frac{\pi}{2}\right)$.

Reading a transformed graph

For $y = 3\sin x$ the amplitude is 3, so the range is $[-3, 3]$; the period (2π) and midline ($y = 0$) are unchanged. Multiplying the whole function stretches it vertically – it does not change how often the wave repeats.

2 Practice

2.1 State the period of $y = \sin x$. [1]

2.2 State the range of $y = \cos x$. [1]

2.3 State the amplitude and midline of $y = \sin x$. [2]

3 Exam-style questions

3.1 The period of $y = \sin x$ is [1]

- **A** π
- **B** 2π
- **C** $\frac{\pi}{2}$
- **D** 1

3.2 The range of $y = \cos x$ is [1]

- **A** all real numbers
- **B** $[0, 1]$
- **C** $[-1, 1]$
- **D** $[-2, 2]$

3.3 $y = \sin x$.

(a) State its maximum value. [1]

(b) State its minimum value. [1]

(c) State the smallest $x > 0$ where the maximum occurs. [1]

Solutions

2.1 2π.

2.2 $[-1, 1]$.

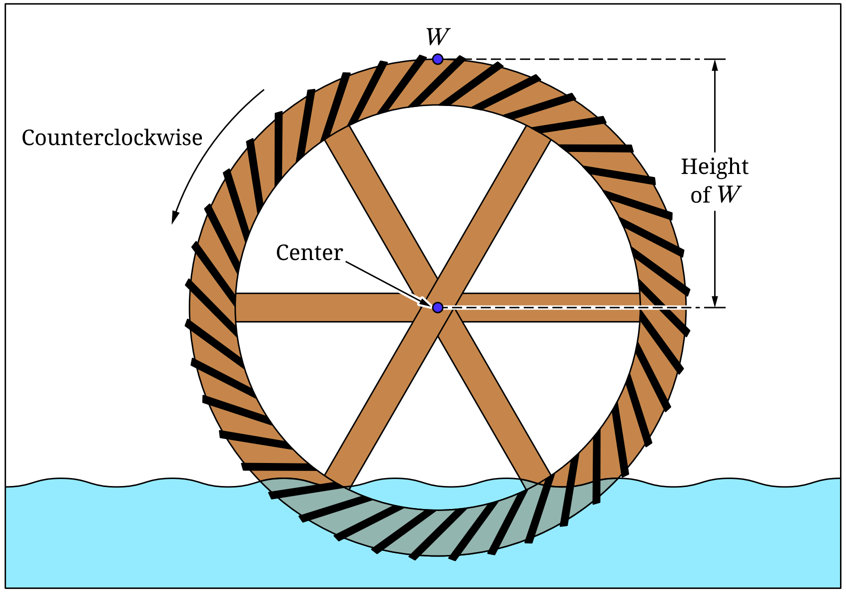
2.3 amplitude 1; midline $y = 0$.

3.1 **B**.

3.2 **C**.

3.3 (a) 1. (b) -1. (c) $x = \frac{\pi}{2}$.

3. The figure shows a circular waterwheel that rotates in a counterclockwise direction at a constant rate and completes 1 revolution in 10 seconds. At time $t = 0$, point W on the edge of the waterwheel is at a height of 6 feet directly above the center of the waterwheel. The height of point W from the horizontal line through the center of the waterwheel periodically decreases and increases as the waterwheel moves.

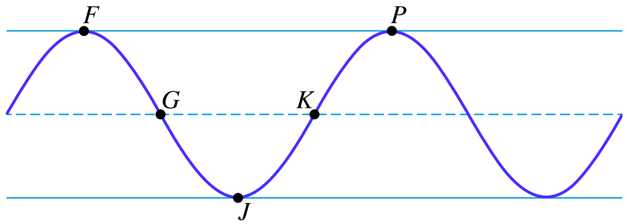


Note: Figure not drawn to scale.

The sinusoidal function h models the height of point W from the horizontal line through the center of the waterwheel, in feet, as a function of time t, in seconds. A positive value of $h(t)$ indicates W is above the line through the center; a negative value of $h(t)$ indicates W is below the line through the center.

A. The graph of h and its dashed midline for two full cycles is shown. Five points, F, G, J, K, and P, are labeled on the graph. No scale is indicated, and no axes are presented.

Determine possible coordinates $(t, h(t))$ for the five points: F, G, J, K, and P.



B. The function h can be written in the form $h(t) = a\sin(b(t+c)) + d$. Find values of constants a, b, c, and d.

C. Refer to the graph of h in part A. The t-coordinate of J is t_1, and the t-coordinate of K is t_2.

i. On the interval (t_1, t_2), which of the following is true about h?

a. h is positive and increasing.

b. h is positive and decreasing.

c. h is negative and increasing.

d. h is negative and decreasing.

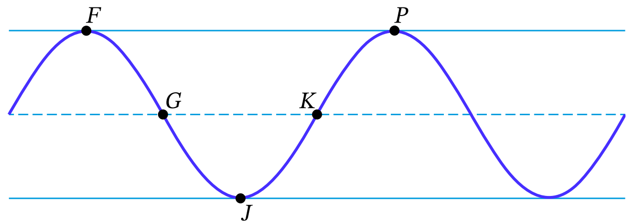
ii. On the interval (t_1, t_2), describe the concavity of the graph of h and determine whether the rate of change of h is increasing or decreasing.

3. For a guitar to make a sound, the strings need to vibrate, or move up and down or back and forth, in a motion that can be modeled by a periodic function.

At time $t = 0$ seconds, point X on one vibrating guitar string starts at its highest position, 2 millimeters above its resting position. Then it passes through its resting position and moves to its lowest position, 2 millimeters below the resting position. Point X then passes through its resting position and returns to 2 millimeters above the resting position. This motion occurs 200 times in 1 second.

The sinusoidal function h models how far point X is from its resting position, in millimeters, as a function of time t, in seconds. A positive value of $h(t)$ indicates the point is above the resting position; a negative value of $h(t)$ indicates the point is below the resting position.

A. The graph of h and its dashed midline for two full cycles is shown. Five points, F, G, J, K, and P, are labeled on the graph. No scale is indicated, and no axes are presented.
Determine possible coordinates $(t, h(t))$ for the five points: F, G, J, K, and P.



B. The function h can be written in the form $h(t) = a\sin\big(b(t+c)\big) + d$. Find values of constants a, b, c, and d.

C. Refer to the graph of h in part A. The t-coordinate of G is t_1, and the t-coordinate of J is t_2.

i. On the interval (t_1, t_2), which of the following is true about h?

a. h is positive and increasing.

b. h is positive and decreasing.

c. h is negative and increasing.

d. h is negative and decreasing.

ii. On the interval (t_1, t_2), describe the concavity of the graph of h and determine whether the rate of change of h is increasing or decreasing.

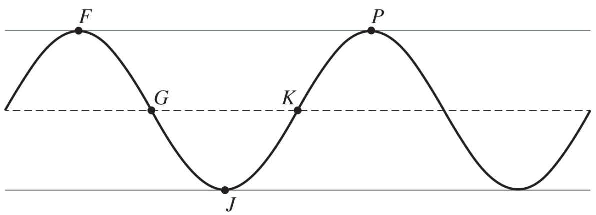
Note: Figure not drawn to scale.

3. The tire of a car has a radius of 9 inches, and a person rolls the tire forward at a constant rate on level ground, as shown in the figure. Point W on the edge of the tire touches the ground at time $t = \frac{1}{2}$ second. The tire completes a full rotation, and the next time W touches the ground is at time $t = \frac{5}{2}$ seconds. The maximum height of W above the ground is 18 inches. As the tire rolls, the height of W above the ground periodically increases and decreases.

The sinusoidal function h models the height of point W above the ground, in inches, as a function of time t, in seconds.

(A) The graph of h and its dashed midline for two full cycles is shown. Five points, F, G, J, K, and P, are labeled on the graph. No scale is indicated, and no axes are presented.

Determine possible coordinates $(t, h(t))$ for the five points: F, G, J, K, and P.



(B) The function h can be written in the form $h(t) = a\sin(b(t + c)) + d$. Find values of constants a, b, c, and d.

(C) Refer to the graph of h in part (A). The t-coordinate of K is t_1, and the t-coordinate of P is t_2.

(i) On the interval (t_1, t_2), which of the following is true about h ?

a. h is positive and increasing.

b. h is positive and decreasing.

c. h is negative and increasing.

d. h is negative and decreasing.

(ii) Describe how the rate of change of h is changing on the interval (t_1, t_2).

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

3.5 Sinusoidal Functions

Name: ______ Class: ______ Date: ______ **Total: 9 marks**

KEY WORDS sinusoidal function 正弦型函数 • amplitude 振幅 • midline 中线 • shift 相移 • period 周期

Objective

Build the skills to answer exam questions on **sinusoidal functions**.

You must be able to:

- read the amplitude, period, phase shift, and midline from $y = a\sin(b(x-h)) + k$

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ **The general sinusoid**

$$y = a\sin\big(b(x-h)\big) + k.$$

- $|a|$ = **amplitude** 振幅;
- period $= \dfrac{2\pi}{b}$;
- h = horizontal (**phase**) shift 相移;
- k = **midline** 中线 (vertical shift).

■ **Example**

$y = 3\sin(2x) + 1$: amplitude 3, period $\frac{2\pi}{2} = \pi$, midline $y = 1$.

2 Practice

2.1 In $y = a\sin(bx)$, state the amplitude. [1]

2.2 State the period of $y = \sin(2x)$. [2]

2.3 In $y = \sin x + 3$, state the midline. [1]

3 Exam-style questions

3.1 In $y = a\sin(b(x-h)) + k$, the amplitude is [1]

- **A** b
- **B** $|a|$
- **C** k
- **D** h

3.2 The period of $y = a\sin(bx)$ is [1]

- **A** 2π
- **B** $\dfrac{2\pi}{b}$
- **C** $\dfrac{b}{2\pi}$
- **D** πb

3.3 $y = 3\sin(2x) + 1$.

(a) State the amplitude. [1]

(b) State the period. [1]

(c) State the midline. [1]

Solutions

2.1 $|a|$.

2.2 period $= \dfrac{2\pi}{2} = \pi$.

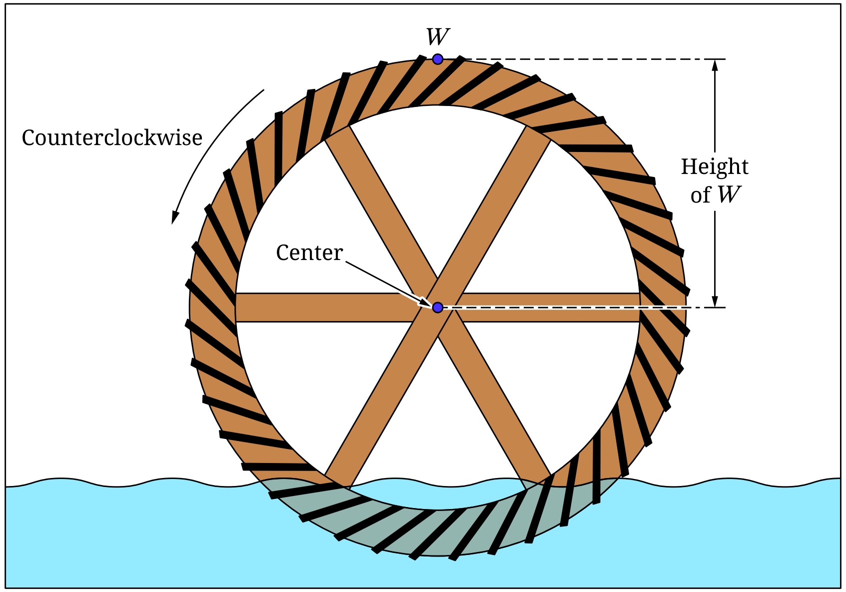
2.3 $y = 3$.

3.1 **B**.

3.2 **B**.

3.3 (a) 3. (b) π. (c) $y = 1$.

3. The figure shows a circular waterwheel that rotates in a counterclockwise direction at a constant rate and completes 1 revolution in 10 seconds. At time $t = 0$, point W on the edge of the waterwheel is at a height of 6 feet directly above the center of the waterwheel. The height of point W from the horizontal line through the center of the waterwheel periodically decreases and increases as the waterwheel moves.

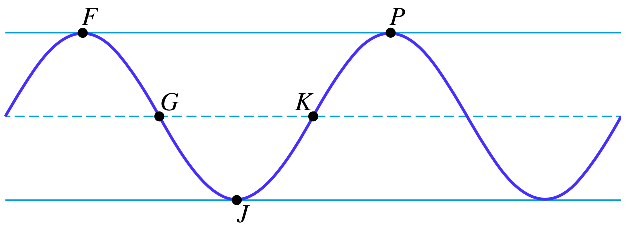


Note: Figure not drawn to scale.

The sinusoidal function h models the height of point W from the horizontal line through the center of the waterwheel, in feet, as a function of time t, in seconds. A positive value of $h(t)$ indicates W is above the line through the center; a negative value of $h(t)$ indicates W is below the line through the center.

A. The graph of h and its dashed midline for two full cycles is shown. Five points, F, G, J, K, and P, are labeled on the graph. No scale is indicated, and no axes are presented.

Determine possible coordinates $(t, h(t))$ for the five points: F, G, J, K, and P.



B. The function h can be written in the form $h(t) = a\sin(b(t+c)) + d$. Find values of constants a, b, c, and d.

C. Refer to the graph of h in part A. The t-coordinate of J is t_1, and the t-coordinate of K is t_2.

i. On the interval (t_1, t_2), which of the following is true about h?

a. h is positive and increasing.

b. h is positive and decreasing.

c. h is negative and increasing.

d. h is negative and decreasing.

ii. On the interval (t_1, t_2), describe the concavity of the graph of h and determine whether the rate of change of h is increasing or decreasing.

3. For a guitar to make a sound, the strings need to vibrate, or move up and down or back and forth, in a motion that can be modeled by a periodic function.

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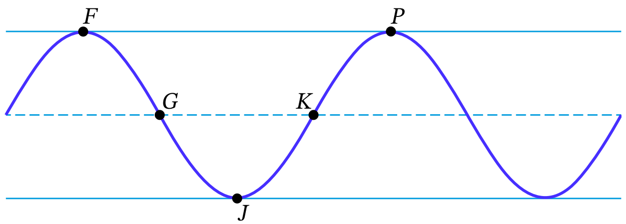
The sinusoidal function h models how far point X is from its resting position, in millimeters, as a function of time t, in seconds. A positive value of $h(t)$ indicates the point is above the resting position; a negative value of $h(t)$ indicates the point is below the resting position.

A. The graph of h and its dashed midline for two full cycles is shown. Five points, F, G, J, K, and P, are labeled on the graph. No scale is indicated, and no axes are presented.
Determine possible coordinates $(t, h(t))$ for the five points: F, G, J, K, and P.



B. The function h can be written in the form $h(t) = a\sin(b(t+c)) + d$. Find values of constants a, b, c, and d.

C. Refer to the graph of h in part A. The t-coordinate of G is t_1, and the t-coordinate of J is t_2.

i. On the interval (t_1, t_2), which of the following is true about h?

a. h is positive and increasing.

b. h is positive and decreasing.

c. h is negative and increasing.

d. h is negative and decreasing.

ii. On the interval (t_1, t_2), describe the concavity of the graph of h and determine whether the rate of change of h is increasing or decreasing.

AP PRECALCULUS • EXERCISE SHEET

3.6 Sinusoidal Function Transformations

Name: ____________ Class: ________ Date: __________ **Total: 9 marks**

KEY WORDS period 周期 • sinusoidal function 正弦型函数 • amplitude 振幅 • midline 中线

Objective

Build the skills to answer exam questions on **sinusoidal function transformations**.

You must be able to:

- describe how a, b, h, and k transform $y = \sin x$

1 Worked examples

Study these first. Each one shows the method for a question type used later.

Transformations of a sinusoid

Starting from $y = \sin x$, in $y = a\sin(b(x-h)) + k$:

- a scales vertically (sets the amplitude);
- b scales horizontally (period $= \frac{2\pi}{b}$);
- h shifts horizontally;
- k shifts vertically (sets the midline).

Example

$y = 2\sin(x) - 3$: amplitude 2, midline $y = -3$.

2 Practice

2.1 State the effect of a in $y = a\sin x$. [1]

2.2 State the effect of k in $y = \sin x + k$. [1]

2.3 For $y = 2\sin(x) - 3$, state the amplitude and midline. [2]

3 Exam-style questions

3.1 In $y = a\sin(bx)$, increasing b [1]

- **A** raises the midline
- **B** shortens the period
- **C** raises the amplitude
- **D** shifts the graph right

3.2 The $+k$ in $y = \sin x + k$ [1]

- **A** changes the period
- **B** shifts the midline
- **C** changes the amplitude
- **D** reflects the graph

3.3 $y = 4\sin(x) + 2$.

(a) State the amplitude. [1]

(b) State the midline. [1]

(c) State the maximum value. [1]

Solutions

2.1 it scales the graph vertically, setting the amplitude to $|a|$.

2.2 it shifts the graph vertically, moving the midline to $y = k$.

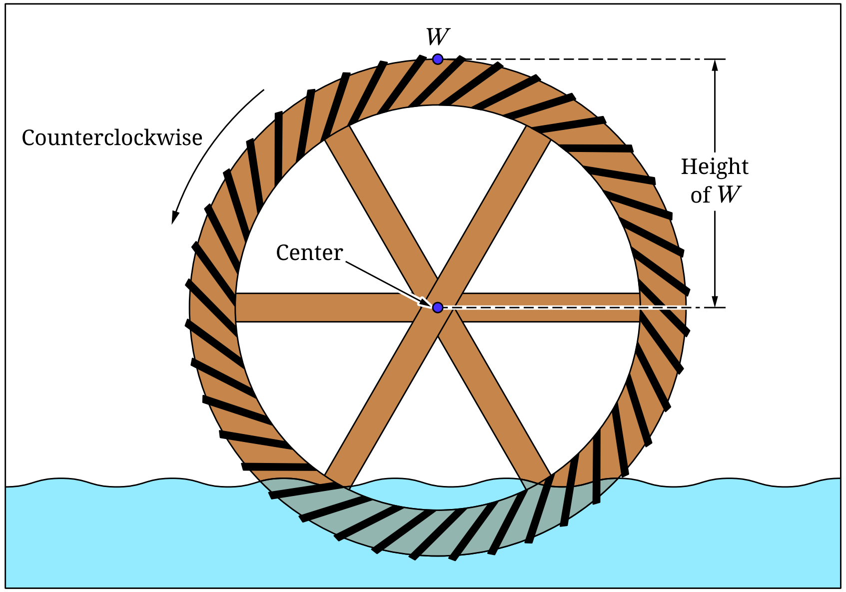
2.3 amplitude 2; midline $y = -3$.

3.1 B.

3.2 B.

3.3 (a) 4. (b) $y = 2$. (c) $2 + 4 = 6$.

3. The figure shows a circular waterwheel that rotates in a counterclockwise direction at a constant rate and completes 1 revolution in 10 seconds. At time $t = 0$, point W on the edge of the waterwheel is at a height of 6 feet directly above the center of the waterwheel. The height of point W from the horizontal line through the center of the waterwheel periodically decreases and increases as the waterwheel moves.

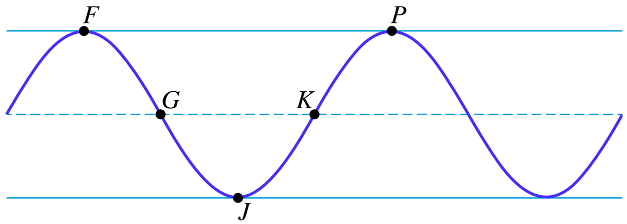


Note: Figure not drawn to scale.

The sinusoidal function h models the height of point W from the horizontal line through the center of the waterwheel, in feet, as a function of time t, in seconds. A positive value of $h(t)$ indicates W is above the line through the center; a negative value of $h(t)$ indicates W is below the line through the center.

A. The graph of h and its dashed midline for two full cycles is shown. Five points, F, G, J, K, and P, are labeled on the graph. No scale is indicated, and no axes are presented.

Determine possible coordinates $(t, h(t))$ for the five points: F, G, J, K, and P.



B. The function h can be written in the form $h(t) = a\sin(b(t+c)) + d$. Find values of constants a, b, c, and d.

C. Refer to the graph of h in part A. The t-coordinate of J is t_1, and the t-coordinate of K is t_2.

i. On the interval (t_1, t_2), which of the following is true about h?

a. h is positive and increasing.

b. h is positive and decreasing.

c. h is negative and increasing.

d. h is negative and decreasing.

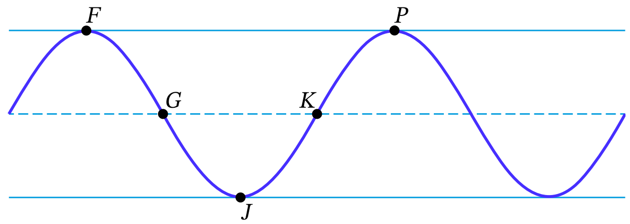
ii. On the interval (t_1, t_2), describe the concavity of the graph of h and determine whether the rate of change of h is increasing or decreasing.

3. For a guitar to make a sound, the strings need to vibrate, or move up and down or back and forth, in a motion that can be modeled by a periodic function.

At time $t = 0$ seconds, point X on one vibrating guitar string starts at its highest position, 2 millimeters above its resting position. Then it passes through its resting position and moves to its lowest position, 2 millimeters below the resting position. Point X then passes through its resting position and returns to 2 millimeters above the resting position. This motion occurs 200 times in 1 second.

The sinusoidal function h models how far point X is from its resting position, in millimeters, as a function of time t, in seconds. A positive value of $h(t)$ indicates the point is above the resting position; a negative value of $h(t)$ indicates the point is below the resting position.

A. The graph of h and its dashed midline for two full cycles is shown. Five points, F, G, J, K, and P, are labeled on the graph. No scale is indicated, and no axes are presented.
Determine possible coordinates $(t, h(t))$ for the five points: F, G, J, K, and P.



B. The function h can be written in the form $h(t) = a\sin(b(t+c)) + d$. Find values of constants a, b, c, and d.

C. Refer to the graph of h in part A. The t-coordinate of G is t_1, and the t-coordinate of J is t_2.

i. On the interval (t_1, t_2), which of the following is true about h?

a. h is positive and increasing.

b. h is positive and decreasing.

c. h is negative and increasing.

d. h is negative and decreasing.

ii. On the interval (t_1, t_2), describe the concavity of the graph of h and determine whether the rate of change of h is increasing or decreasing.

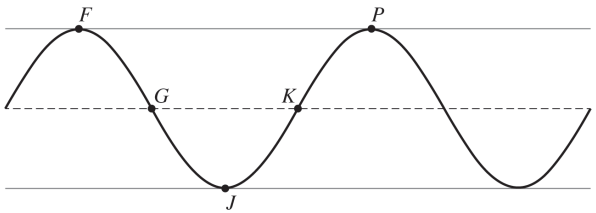
Note: Figure not drawn to scale.

3. The tire of a car has a radius of 9 inches, and a person rolls the tire forward at a constant rate on level ground, as shown in the figure. Point W on the edge of the tire touches the ground at time $t = \frac{1}{2}$ second. The tire completes a full rotation, and the next time W touches the ground is at time $t = \frac{5}{2}$ seconds. The maximum height of W above the ground is 18 inches. As the tire rolls, the height of W above the ground periodically increases and decreases.

The sinusoidal function h models the height of point W above the ground, in inches, as a function of time t, in seconds.

(A) The graph of h and its dashed midline for two full cycles is shown. Five points, F, G, J, K, and P, are labeled on the graph. No scale is indicated, and no axes are presented.

Determine possible coordinates $(t, h(t))$ for the five points: F, G, J, K, and P.



(B) The function h can be written in the form $h(t) = a\sin(b(t + c)) + d$. Find values of constants a, b, c, and d.

(C) Refer to the graph of h in part (A). The t-coordinate of K is t_1, and the t-coordinate of P is t_2.

(i) On the interval (t_1, t_2), which of the following is true about h ?

a. h is positive and increasing.

b. h is positive and decreasing.

c. h is negative and increasing.

d. h is negative and decreasing.

(ii) Describe how the rate of change of h is changing on the interval (t_1, t_2).

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

AP PRECALCULUS • EXERCISE SHEET

3.7 Sinusoidal Function Context and Data Modeling

Name: ____________ Class: ________ Date: __________ **Total: 9 marks**

KEY WORDS period 周期 • midline 中线 • amplitude 振幅 • sinusoidal function 正弦型函数

Objective

Build the skills to answer exam questions on **sinusoidal function context and data modeling**.

You must be able to:

- build a sinusoidal model from a maximum, minimum, and **period** 周期

1 Worked examples

Study these first. Each one shows the method for a question type used later.

Building a sinusoid from data

- amplitude $= \dfrac{\max - \min}{2}$;
- midline $= \dfrac{\max + \min}{2}$;
- $b = \dfrac{2\pi}{\text{period}}$.

Example

A tide with max 6 m, min 2 m, period 12 h: amplitude 2, midline 4, $b = \frac{2\pi}{12} = \frac{\pi}{6}$, giving $y = 2\sin\left(\frac{\pi}{6}t\right) + 4$.

2 Practice

2.1 State the formula for b given the period. [1]

2.2 A quantity has max 10 and min 4. Find the amplitude and midline. [2]

2.3 State b for a period of 24. [1]

3 Exam-style questions

3.1 Given period P, the value b equals [1]

- **A** $P/2\pi$
- **B** $2\pi/P$
- **C** πP
- **D** $2\pi P$

3.2 The midline of a sinusoidal model equals [1]

- **A** $\dfrac{\max - \min}{2}$
- **B** $\dfrac{\max + \min}{2}$
- **C** max
- **D** min

3.3 A Ferris wheel has maximum height 50 m, minimum 2 m, and period 60 s.

(a) Find the amplitude. [1]

(b) Find the midline. [1]

(c) Find the value of b. [1]

Solutions

2.1 $b = \dfrac{2\pi}{\text{period}}$.

2.2 amplitude $= \dfrac{10-4}{2} = 3$; midline $= \dfrac{10+4}{2} = 7$.

2.3 $b = \dfrac{2\pi}{24} = \dfrac{\pi}{12}$.

3.1 **B**.

3.2 **B**.

3.3 (a) $\dfrac{50-2}{2} = 24$ m. (b) $\dfrac{50+2}{2} = 26$ m. (c) $b = \dfrac{2\pi}{60} = \dfrac{\pi}{30}$.

AP PRECALCULUS • EXERCISE SHEET

3.8 The Tangent Function

Name: ____________ Class: ________ Date: __________ **Total: 9 marks**

KEY WORDS vertical asymptotes 垂直渐近线 • period 周期

Objective

Build the skills to answer exam questions on **the tangent function**.

You must be able to:

- state the period and **vertical asymptotes 垂直渐近线** of $y = \tan x$
- evaluate the tangent at simple angles

1 Worked examples

Study these first. Each one shows the method for a question type used later.

The tangent function

$\tan x = \dfrac{\sin x}{\cos x}$.

- period π;
- **vertical asymptotes** where $\cos x = 0$, i.e. $x = \frac{\pi}{2} + n\pi$;
- increasing on each branch.

Example

$\tan 0 = 0$, $\tan \frac{\pi}{4} = 1$, and $\tan \frac{\pi}{2}$ is undefined.

2 Practice

2.1 State the period of $y = \tan x$. [1]

2.2 State where $\tan x$ has vertical asymptotes. [1]

2.3 Evaluate $\tan 0$ and $\tan \frac{\pi}{4}$. [2]

3 Exam-style questions

3.1 The period of $y = \tan x$ is [1]

- **A** π
- **B** 2π
- **C** $\frac{\pi}{2}$
- **D** 4π

3.2 $y = \tan x$ has vertical asymptotes where [1]

- **A** $\sin x = 0$
- **B** $\cos x = 0$
- **C** $\tan x = 0$
- **D** $x = 0$

3.3 Evaluate each.

(a) $\tan 0$. [1]

(b) $\tan \frac{\pi}{4}$. [1]

(c) State whether $\tan \frac{\pi}{2}$ is defined. [1]

Solutions

2.1 π.

2.2 where $\cos x = 0$, i.e. $x = \frac{\pi}{2} + n\pi$.

2.3 $\tan 0 = 0$; $\tan \frac{\pi}{4} = 1$.

3.1 **A**.

3.2 **B**.

3.3 (a) 0. (b) 1. (c) undefined.

3.9 Inverse Trigonometric Functions

Name: ____________ Class: ________ Date: __________ **Total: 9 marks**

KEY WORDS inverse trigonometric functions 反三角函数 • ranges 值域

Objective

Build the skills to answer exam questions on **inverse trigonometric functions**.

You must be able to:

- describe what arcsin, arccos, and arctan return
- state their restricted **ranges** 值域

1 Worked examples

Study these first. Each one shows the method for a question type used later.

Inverse trig functions

- $\arcsin(x) =$ the angle whose sine is x; range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$;
- $\arccos(x)$; range $[0, \pi]$;
- $\arctan(x)$; range $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Each inverts its trig function on a restricted domain.

Example

$\arcsin(1) = \frac{\pi}{2}$, $\arccos(1) = 0$.

2 Practice

2.1 State what $\arcsin(x)$ returns. [1]

2.2 State the range of $\arctan(x)$. [1]

2.3 Evaluate $\arcsin(1)$ and $\arccos(1)$. [2]

3 Exam-style questions

3.1 $\arcsin(x)$ gives [1]

- **A** the sine of x
- **B** the angle whose sine is x
- **C** $1/\sin x$
- **D** $\sin(1/x)$

3.2 The range of $\arccos(x)$ is [1]

- **A** $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
- **B** $[0, \pi]$
- **C** all real numbers
- **D** $[0, 2\pi]$

3.3 Evaluate each.

(a) $\arcsin(0)$. [1]

(b) $\arccos(0)$. [1]

(c) $\arctan(1)$. [1]

Solutions

2.1 the angle whose sine is x.

2.2 $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

2.3 $\arcsin(1) = \frac{\pi}{2}$; $\arccos(1) = 0$.

3.1 **B**.

3.2 **B**.

3.3 (a) 0. (b) $\frac{\pi}{2}$. (c) $\frac{\pi}{4}$.

4. **Directions:**

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

(A) The functions g and h are given by

$$g(x) = e^{(x+3)}$$
$$h(x) = \arcsin\left(\frac{x}{2}\right).$$

(i) Solve $g(x) = 10$ for values of x in the domain of g.

(ii) Solve $h(x) = \frac{\pi}{4}$ for values of x in the domain of h.

(B) The functions j and k are given by

$$j(x) = \log_{10}(8x^5) + \log_{10}(2x^2) - 9\log_{10}x$$
$$k(x) = \left(\frac{1 - \sin^2 x}{\sin x}\right)\sec x.$$

(i) Rewrite $j(x)$ as a single logarithm base 10 without negative exponents in any part of the expression. Your result should be of the form $\log_{10}$(expression).

(ii) Rewrite $k(x)$ as a single term involving $\tan x$.

(C) The function m is given by

$$m(x) = \cos^{-1}(\tan(2x)).$$

Find all values in the domain of m that yield an output value of 0.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

STOP

END OF EXAM

AP PRECALCULUS • EXERCISE SHEET

3.10 Trigonometric Equations and Inequalities

Name: ______ Class: ______ Date: ______ **Total: 9 marks**

KEY WORDS periodicity 周期性 • period 周期

Objective

Build the skills to answer exam questions on **trigonometric equations and inequalities**.

You must be able to:

- solve a simple trigonometric equation on a given interval
- explain why such equations have infinitely many solutions

1 Worked examples

Study these first. Each one shows the method for a question type used later.

> **Solving trig equations**
>
> Solve $\sin x = c$ by finding one solution, then using symmetry and **periodicity** 周期性 to get the rest ($+2\pi n$).
>
> **Example**
>
> $\sin x = \frac{1}{2}$ gives $x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$ on $[0, 2\pi)$, plus $2\pi n$ for all solutions.

2 Practice

2.1 Solve $\sin x = 0$ on $[0, 2\pi)$. [1]

2.2 State why a trig equation has infinitely many solutions. [1]

2.3 Solve $\cos x = 1$ on $[0, 2\pi)$. [2]

3 Exam-style questions

3.1 The equation $\sin x = \frac{1}{2}$ has [1]

- **A** one solution in total
- **B** two solutions in $[0, 2\pi)$
- **C** no solution
- **D** no solution in $[0, 2\pi)$

3.2 Trig equations have infinitely many solutions because sine and cosine are [1]

- **A** linear
- **B** periodic
- **C** polynomial
- **D** rational

3.3 Solve $\sin x = 1$.

(a) State the solution on $[0, 2\pi)$. [1]

(b) State the general solution (all x). [1]

(c) State the amount added each time. [1]

Solutions

2.1 $x = 0$ and $x = \pi$.

2.2 sine and cosine are periodic, so each solution repeats every 2π.

2.3 $\cos x = 1$ at $x = 0$ only on $[0, 2\pi)$.

3.1 **B**.

3.2 **B**.

3.3 (a) $x = \frac{\pi}{2}$. (b) $x = \frac{\pi}{2} + 2\pi n$. (c) 2π.

4. Directions:

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

A. The functions g and h are given by

$$g(x) = e^{2x}$$
$$h(x) = \log_2(5x)$$

i. Solve $g(x) = \frac{1}{e^6}$ for values of x in the domain of g.

ii. Solve $h(x) = 3$ for values of x in the domain of h.

B. The functions j and k are given by

$$j(x) = 7^{(3x+1)} \cdot 7^x$$
$$k(x) = \sin(2x)\sec x$$

i. Rewrite $j(x)$ as an expression of the form $7^{(\text{expression})}$.

ii. Rewrite $k(x)$ as an expression in which $\sin x$ appears exactly once and no other trigonometric functions are involved.

C. The function m is given by $m(x) = \tan^2(3x)$.

Find all input values for m in the interval $\left[0, \frac{\pi}{2}\right]$ that yield an output value of 1.

STOP

END OF EXAM

4. Directions:

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

A. The functions g and h are given by

$$g(x) = 2\log_3 x$$
$$h(x) = 4\cos^2 x$$

i. Solve $g(x) = 4$ for values of x in the domain of g.

ii. Solve $h(x) = 3$ for values of x in the interval $\left[0, \frac{\pi}{2}\right)$.

B. The functions j and k are given by

$$j(x) = \log_2 x + 3\log_2 2$$
$$k(x) = \frac{6}{\tan x(\csc^2 x - 1)}$$

i. Rewrite $j(x)$ as a single logarithm base 2 without negative exponents in any part of the expression. Your result should be of the form $\log_2(\text{expression})$.

ii. Rewrite $k(x)$ as an expression in which $\tan x$ appears exactly once and no other trigonometric functions are involved.

C. The function m is given by $m(x) = e^{2x} - e^x - 12$. Find all input values in the domain of m that yield an output value of 0.

STOP

END OF EXAM

4. **Directions:**

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

(A) The functions g and h are given by

$$g(x) = e^{(x+3)}$$
$$h(x) = \arcsin\left(\frac{x}{2}\right).$$

(i) Solve $g(x) = 10$ for values of x in the domain of g.

(ii) Solve $h(x) = \frac{\pi}{4}$ for values of x in the domain of h.

(B) The functions j and k are given by

$$j(x) = \log_{10}(8x^5) + \log_{10}(2x^2) - 9\log_{10}x$$
$$k(x) = \left(\frac{1 - \sin^2 x}{\sin x}\right)\sec x.$$

(i) Rewrite $j(x)$ as a single logarithm base 10 without negative exponents in any part of the expression. Your result should be of the form $\log_{10}(\text{expression})$.

(ii) Rewrite $k(x)$ as a single term involving $\tan x$.

(C) The function m is given by

$$m(x) = \cos^{-1}(\tan(2x)).$$

Find all values in the domain of m that yield an output value of 0.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

STOP

END OF EXAM

3.11 The Secant, Cosecant, and Cotangent Functions

Name: ____________ Class: ________ Date: __________ **Total: 9 marks**

KEY WORDS reciprocal functions 倒数函数

Objective

Build the skills to answer exam questions on **the secant, cosecant, and cotangent functions**.

You must be able to:

- write sec, csc, and cot as **reciprocal functions 倒数函数** of cos, sin, and tan

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ **Reciprocal functions**

$$\sec x = \frac{1}{\cos x}, \quad \csc x = \frac{1}{\sin x}, \quad \cot x = \frac{\cos x}{\sin x} = \frac{1}{\tan x}.$$

Each is undefined where its denominator is 0.

■ **Example**

$\sec 0 = \dfrac{1}{\cos 0} = 1$; $\csc \frac{\pi}{2} = \dfrac{1}{\sin(\pi/2)} = 1$.

2 Practice

2.1 State $\sec x$ in terms of $\cos x$. [1]

2.2 State $\csc x$ in terms of $\sin x$. [1]

2.3 Evaluate $\sec 0$ and $\csc \frac{\pi}{2}$. [2]

3 Exam-style questions

3.1 $\sec x$ equals [1]

- **A** $1/\sin x$
- **B** $1/\cos x$
- **C** $1/\tan x$
- **D** $\cos x$

3.2 $\cot x$ equals [1]

- **A** $\sin x/\cos x$
- **B** $\cos x/\sin x$
- **C** $1/\sin x$
- **D** $\tan x$

3.3 Answer each.

(a) Evaluate $\sec 0$. [1]

(b) Evaluate $\csc \frac{\pi}{2}$. [1]

(c) State where $\csc x$ is undefined. [1]

Solutions

2.1 $\sec x = 1/\cos x$.

2.2 $\csc x = 1/\sin x$.

2.3 $\sec 0 = 1$; $\csc \frac{\pi}{2} = 1$.

3.1 **B**.

3.2 **B**.

3.3 (a) 1. (b) 1. (c) where $\sin x = 0$ (i.e. $x = n\pi$).

3.12 Equivalent Representations of Trigonometric Functions

Name: ____________ Class: __________ Date: ____________ **Total: 9 marks**

KEY WORDS pythagorean identity 勾股恒等式

Objective

Build the skills to answer exam questions on **equivalent representations of trigonometric functions**.

You must be able to:

- apply the **Pythagorean identity** 勾股恒等式 $\sin^2 x + \cos^2 x = 1$
- use the even/odd symmetry of sine and cosine

1 Worked examples

Study these first. Each one shows the method for a question type used later.

Key identities

- **Pythagorean**: $\sin^2 x + \cos^2 x = 1$, so $\cos^2 x = 1 - \sin^2 x$;
- **even/odd**: $\cos(-x) = \cos x$ (even), $\sin(-x) = -\sin x$ (odd).

Example

If $\sin x = \frac{3}{5}$, then $\cos^2 x = 1 - \frac{9}{25} = \frac{16}{25}$, so $\cos x = \pm\frac{4}{5}$.

2 Practice

2.1 State the Pythagorean identity. [1]

2.2 Rewrite $1 - \cos^2 x$. [1]

2.3 State $\sin(-x)$ in terms of $\sin x$. [2]

3 Exam-style questions

3.1 The Pythagorean identity is [1]

- **A** $\sin x + \cos x = 1$
- **B** $\sin^2 x + \cos^2 x = 1$
- **C** $\sin^2 x - \cos^2 x = 1$
- **D** $\sin x \cdot \cos x = 1$

3.2 $\cos(-x)$ equals [1]

- **A** $-\cos x$
- **B** $\cos x$
- **C** $\sin x$
- **D** $-\sin x$

3.3 Given $\sin x = \frac{3}{5}$.

(a) State $\sin^2 x$. [1]

(b) Use the identity to find $\cos^2 x$. [1]

(c) State a possible value of $\cos x$. [1]

Solutions

2.1 $\sin^2 x + \cos^2 x = 1$.

2.2 $\sin^2 x$.

2.3 $\sin(-x) = -\sin x$.

3.1 **B**.

3.2 **B**.

3.3 (a) $\frac{9}{25}$. (b) $1 - \frac{9}{25} = \frac{16}{25}$. (c) $\frac{4}{5}$ (or $-\frac{4}{5}$).

4. Directions:

- Unless otherwise specified, the domain of a function *f* is assumed to be the set of all real numbers *x* for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

A. The functions *g* and *h* are given by

$$g(x) = e^{2x}$$
$$h(x) = \log_2(5x)$$

i. Solve $g(x) = \frac{1}{e^6}$ for values of *x* in the domain of *g*.

ii. Solve $h(x) = 3$ for values of *x* in the domain of *h*.

B. The functions *j* and *k* are given by

$$j(x) = 7^{(3x+1)} \cdot 7^x$$
$$k(x) = \sin(2x)\sec x$$

i. Rewrite $j(x)$ as an expression of the form $7^{(\text{expression})}$.

ii. Rewrite $k(x)$ as an expression in which $\sin x$ appears exactly once and no other trigonometric functions are involved.

C. The function *m* is given by $m(x) = \tan^2(3x)$.

Find all input values for *m* in the interval $\left[0, \frac{\pi}{2}\right]$ that yield an output value of 1.

STOP

END OF EXAM

4. Directions:

- Unless otherwise specified, the domain of a function *f* is assumed to be the set of all real numbers *x* for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

A. The functions *g* and *h* are given by

$$g(x) = 2\log_3 x$$
$$h(x) = 4\cos^2 x$$

i. Solve $g(x) = 4$ for values of *x* in the domain of *g*.

ii. Solve $h(x) = 3$ for values of *x* in the interval $\left[0, \frac{\pi}{2}\right)$.

B. The functions *j* and *k* are given by

$$j(x) = \log_2 x + 3\log_2 2$$
$$k(x) = \frac{6}{\tan x(\csc^2 x - 1)}$$

i. Rewrite $j(x)$ as a single logarithm base 2 without negative exponents in any part of the expression. Your result should be of the form $\log_2(\text{expression})$.

ii. Rewrite $k(x)$ as an expression in which $\tan x$ appears exactly once and no other trigonometric functions are involved.

C. The function *m* is given by $m(x) = e^{2x} - e^x - 12$. Find all input values in the domain of *m* that yield an output value of 0.

STOP

END OF EXAM

4. **Directions:**

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

(A) The functions g and h are given by

$$g(x) = e^{(x+3)}$$
$$h(x) = \arcsin\left(\frac{x}{2}\right).$$

(i) Solve $g(x) = 10$ for values of x in the domain of g.

(ii) Solve $h(x) = \frac{\pi}{4}$ for values of x in the domain of h.

(B) The functions j and k are given by

$$j(x) = \log_{10}(8x^5) + \log_{10}(2x^2) - 9\log_{10}x$$
$$k(x) = \left(\frac{1 - \sin^2 x}{\sin x}\right)\sec x.$$

(i) Rewrite $j(x)$ as a single logarithm base 10 without negative exponents in any part of the expression. Your result should be of the form $\log_{10}(\text{expression})$.

(ii) Rewrite $k(x)$ as a single term involving $\tan x$.

(C) The function m is given by

$$m(x) = \cos^{-1}(\tan(2x)).$$

Find all values in the domain of m that yield an output value of 0.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

STOP

END OF EXAM

AP PRECALCULUS • EXERCISE SHEET

3.13 Trigonometry and Polar Coordinates

Name: ____________ Class: ________ Date: __________ **Total: 9 marks**

KEY WORDS polar 极坐标

Objective

Build the skills to answer exam questions on **trigonometry and polar coordinates**.

You must be able to:

- convert between **polar** 极坐标 (r, θ) and rectangular (x, y) coordinates

1 Worked examples

Study these first. Each one shows the method for a question type used later.

Polar coordinates

A point is (r, θ): r is the distance from the origin, θ the angle. Convert with

$$x = r\cos\theta, \quad y = r\sin\theta, \quad r = \sqrt{x^2 + y^2}, \quad \tan\theta = \frac{y}{x}.$$

Example

$(r, \theta) = (2, 0)$: $x = 2\cos 0 = 2$, $y = 2\sin 0 = 0$, so the point is $(2, 0)$.

2 Practice

2.1 State x in terms of r and θ. [1]

2.2 State the formula for r given x and y. [1]

2.3 Convert $(r, \theta) = (2, 0)$ to rectangular coordinates. [2]

3 Exam-style questions

3.1 In polar coordinates, x equals [1]

- **A** $r\sin\theta$
- **B** $r\cos\theta$
- **C** r/θ
- **D** $r + \theta$

3.2 The radius r equals [1]

- **A** $x + y$
- **B** $\sqrt{x^2 + y^2}$
- **C** $x^2 + y^2$
- **D** x/y

3.3 Convert $(r, \theta) = \left(4, \frac{\pi}{2}\right)$.

(a) Find x. [1]

(b) Find y. [1]

(c) State the rectangular point. [1]

Solutions

2.1 $x = r\cos\theta$.

2.2 $r = \sqrt{x^2 + y^2}$.

2.3 $x = 2\cos 0 = 2$; $y = 2\sin 0 = 0$; point $(2, 0)$.

3.1 **B**.

3.2 **B**.

3.3 (a) $x = 4\cos\frac{\pi}{2} = 0$. (b) $y = 4\sin\frac{\pi}{2} = 4$. (c) $(0, 4)$.

AP PRECALCULUS • EXERCISE SHEET

3.14 Polar Function Graphs

Name: ____________ Class: ________ Date: __________ **Total: 8 marks**

KEY WORDS polar function 极坐标函数

Objective

Build the skills to answer exam questions on **polar function graphs**.

You must be able to:

- interpret a **polar function 极坐标函数** $r = f(\theta)$
- recognise the circle $r =$ constant

1 Worked examples

Study these first. Each one shows the method for a question type used later.

Polar graphs

A polar function $r = f(\theta)$ gives the distance r from the origin at each angle θ.

- $r =$ constant is a **circle** centred at the origin;
- $r = a\cos\theta$ is a circle through the origin;
- $r = a + b\cos\theta$ is a **limaçon**.

Example

$r = 4$ is a circle of radius 4 centred at the origin.

2 Practice

2.1 State what $r = 3$ graphs in polar form. [1]

2.2 State what $r = f(\theta)$ represents. [1]

2.3 For $r = 2$, state the radius of the circle. [1]

3 Exam-style questions

3.1 The polar graph $r = 5$ is [1]

- **A** a line
- **B** a circle of radius 5
- **C** a spiral
- **D** a single point

3.2 In $r = f(\theta)$, r represents the [1]

- **A** angle
- **B** distance from the origin
- **C** slope
- **D** area

3.3 Consider $r = 4$.

(a) State the shape. [1]

(b) State the radius. [1]

(c) State the centre. [1]

Solutions

2.1 a circle of radius 3 centred at the origin.

2.2 the distance from the origin as a function of the angle.

2.3 2.

3.1 **B**.

3.2 **B**.

3.3 (a) a circle. (b) 4. (c) the origin.

AP PRECALCULUS • EXERCISE SHEET

3.15 Rates of Change in Polar Functions

Name: ____________ Class: ________ Date: __________ **Total: 9 marks**

KEY WORDS average rate of change 平均变化率

Objective

Build the skills to answer exam questions on **rates of change in polar functions**.

You must be able to:

- compute the **average rate of change** 平均变化率 of r with respect to θ
- interpret whether the curve spirals in or out

1 Worked examples

Study these first. Each one shows the method for a question type used later.

Rate of change of r

The average rate of change of r over $[\theta_1, \theta_2]$ is

$$\frac{r(\theta_2) - r(\theta_1)}{\theta_2 - \theta_1}.$$

If r **increases** as θ increases, the curve spirals **outward**; if r decreases, it spirals inward.

Example

$r(\theta) = 3\theta$: $r(0) = 0$, $r(2) = 6$; average rate $= \frac{6-0}{2-0} = 3$.

2 Practice

2.1 State the average rate of change of r over $[\theta_1, \theta_2]$. [1]

2.2 State what it means if r increases as θ increases. [1]

2.3 For $r(\theta) = 2\theta$, find $r(0)$ and $r(\pi)$. [2]

3 Exam-style questions

3.1 The average rate of change of r on $[a, b]$ is [1]

- **A** $r(b) \cdot r(a)$
- **B** $\dfrac{r(b) - r(a)}{b - a}$
- **C** $r(b) + r(a)$
- **D** $b - a$

3.2 If r increases as θ increases, the curve [1]

- **A** spirals inward
- **B** spirals outward
- **C** stays a circle
- **D** is a straight line

3.3 $r(\theta) = 3\theta$.

(a) Find $r(0)$. [1]

(b) Find $r(2)$. [1]

(c) Find the average rate of change on $[0, 2]$. [1]

Solutions

2.1 $\dfrac{r(\theta_2) - r(\theta_1)}{\theta_2 - \theta_1}$.

2.2 the curve spirals outward (moves farther from the origin).

2.3 $r(0) = 0$; $r(\pi) = 2\pi$.

3.1 **B**.

3.2 **B**.

3.3 (a) 0. (b) 6. (c) $\frac{6-0}{2-0} = 3$.