Week 8

AP Precalculus

Homework bundle for this week

本周作业合集

exercises.lol

Contents 目录

AP Precalculus Name: ____________ Score: ______

1. An **arithmetic sequence** has a constant…
   - ☐ difference between consecutive terms
   - ☐ ratio between consecutive terms
   - ☐ first term only
   - ☐ number of terms
2. Over equal input intervals, a **linear function** changes by…
   - ☐ adding a constant amount
   - ☐ multiplying by a constant factor
   - ☐ squaring the input
   - ☐ a random amount
3. An **exponential function** has the form…
   - ☐ $f(x) = a\,b^x$ with $b > 0,\ b \neq 1$
   - ☐ $f(x) = ax^2 + bx + c$
   - ☐ $f(x) = ax + b$
   - ☐ $f(x) = \frac{a}{x}$
4. The **product rule** for exponents says $b^m \cdot b^n =$
   - ☐ b^{m+n}
   - ☐ b^{mn}
   - ☐ b^{m-n}
   - ☐ $2b^{m+n}$
5. To build $y = a\,b^x$ from data, the coefficient a is the value when…
   - ☐ $x = 0$
   - ☐ $x = 1$
   - ☐ $y = 0$
   - ☐ the data ends
6. Solve $2^x = 8$ by matching bases.

   ____

7. If a quantity grows by 20% each period, the growth ______ (the base b) is 1.2.

____

8. Given enough time, an exponential model eventually grows faster than any polynomial model.

☐ True ☐ False

9. If $g(x) = x + 1$ and $f(x) = x^2$, what is $f(g(2))$?

____

10. A function has an inverse function only if it is ______ (each output comes from exactly one input).

____

AP Precalculus

1. **difference between consecutive terms**

   Each term is the previous one **plus** a fixed **common difference** — that steady addition makes it arithmetic.

2. **adding a constant amount**

   A line has a constant slope, so equal steps in x add the same amount to y — like an arithmetic sequence.

3. $f(x) = a\,b^x$ **with** $b > 0,\ b \neq 1$

   The variable sits in the **exponent** over a fixed base b — that is what makes it exponential, not polynomial.

4. b^{m+n}

   Multiplying powers of the same base **adds** the exponents: $b^m \cdot b^n = b^{m+n}$.

5. $x = 0$

   At $x = 0$, $b^0 = 1$, so $y = a$: the coefficient is the **initial value**.

6. **3**

   $8 = 2^3$, so $2^x = 2^3$ gives $x = 3$ — equal bases mean equal exponents.

7. **factor**

   A 20% increase multiplies by $1 + 0.20 = 1.2$ — that multiplier is the **growth factor**.

8. **True**

   Exponential growth outpaces x^2, x^3, even x^{100} in the long run — the exponent always wins.

9. **9**

   Inner first: $g(2) = 3$. Then outer: $f(3) = 3^2 = 9$.

10. **one-to-one**

    If two inputs share an output, the reverse map is ambiguous — so only a **one-to-one** function is invertible.

2.1 Change in Arithmetic and Geometric Sequences

Name: ______________ Class: __________ Date: ____________ **Total: 9 marks**

KEY WORDS sequence 数列 • geometric sequence 等比数列 • arithmetic sequence 等差数列 • common difference 公
• common ratio 公比

Objective

Build the skills to answer exam questions on **change in arithmetic and geometric sequences**.

You must be able to:

- use the **common difference 公差** of an arithmetic sequence
- use the **common ratio 公比** of a geometric sequence
- write the n-th term of each

1 Worked examples

Study these first. Each one shows the method for a question type used later.

Arithmetic sequences

A constant **common difference** d is added each step: $a_n = a_1 + (n-1)d$.

Geometric sequences

A constant **common ratio** r is multiplied each step: $g_n = g_1 \cdot r^{n-1}$.

Example

$3, 7, 11, 15, \ldots$ has $d = 4$; $2, 6, 18, 54, \ldots$ has $r = 3$.

2 Practice

2.1 State the common difference of $3, 7, 11, 15, \ldots$ [1]

2.2 State the common ratio of $2, 6, 18, 54, \ldots$ [1]

2.3 Find the 5-th term of an arithmetic sequence with $a_1 = 4$, $d = 3$. [2]

3 Exam-style questions

3.1 An arithmetic sequence has a constant [1]

- **A** ratio
- **B** difference
- **C** product
- **D** sum

3.2 A geometric sequence has a constant [1]

- **A** difference
- **B** ratio
- **C** sum
- **D** second difference

3.3 A geometric sequence has $g_1 = 5$ and $r = 2$.

(a) State the 2nd term. [1]

(b) State the 3rd term. [1]

(c) Write the formula for g_n. [1]

Solutions

2.1 $d = 4$.

2.2 $r = 3$.

2.3 $a_5 = 4 + (5 - 1)(3) = 4 + 12 = 16$.

3.1 **B**.

3.2 **B**.

3.3 (a) 10. (b) 20. (c) $g_n = 5 \cdot 2^{n-1}$.

2.2 Change in Linear and Exponential Functions

Name: ____________ Class: ________ Date: __________ **Total: 8 marks**

KEY WORDS exponential function 指数函数 • linear function 线性函数 • sequence 数列 • arithmetic sequence 等差
• geometric sequence 等比数列

Objective

Build the skills to answer exam questions on **change in linear and exponential functions**.

You must be able to:

- describe how a **linear function 线性函数** changes over equal intervals
- describe how an **exponential function 指数函数** changes over equal intervals

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Linear vs exponential

- a **linear** function changes by equal **differences** over equal intervals (it **adds** a constant);
- an **exponential** function changes by equal **ratios** over equal intervals (it **multiplies** by a constant).

Linear change matches an arithmetic sequence; exponential change matches a geometric sequence.

■ Spotting the pattern in a table

x	**0**	**1**	**2**	**3**
A	3	5	7	9
B	3	6	12	24

A **adds** 2 each step, so it is **linear**; B **multiplies** by 2 each step, so it is **exponential**. Test which stays constant: the difference (linear) or the ratio (exponential).

2 Practice

2.1 State how a linear function changes over equal intervals. [1]

2.2 State how an exponential function changes over equal intervals. [1]

2.3 A function multiplies by 3 each step. Is it linear or exponential? [1]

3 Exam-style questions

3.1 A function that adds a constant each unit is [1]

- **A** exponential
- **B** linear
- **C** quadratic
- **D** rational

3.2 A function that multiplies by a constant each unit is [1]

- **A** linear
- **B** exponential
- **C** constant
- **D** polynomial

3.3 The values $4, 12, 36, 108, \ldots$ are recorded at equal intervals.

(a) Is the change by equal difference or equal ratio? [1]

(b) State the constant. [1]

(c) State the model type. [1]

Solutions

2.1 by equal differences (it adds a constant amount).

2.2 by equal ratios (it multiplies by a constant factor).

2.3 exponential.

3.1 **B**.

3.2 **B**.

3.3 (a) equal ratio. (b) 3. (c) exponential.

2.3 Exponential Functions

Name: ______________ Class: __________ Date: ____________ **Total: 9 marks**

KEY WORDS growth 增长 • initial value 初始值 • decay 衰减 • base 底数 • exponential function 指数函数

Objective

Build the skills to answer exam questions on **exponential functions**.

You must be able to:

- read the **initial value** 初始值 a and **base** 底数 b from $f(x) = a \cdot b^x$
- decide whether the function models **growth** 增长 or **decay** 衰减

1 Worked examples

Study these first. Each one shows the method for a question type used later.

> ■ **Exponential form**
>
> $f(x) = a \cdot b^x$ with base $b > 0$, $b \neq 1$.
>
> - a is the **initial value** — the output at $x = 0$;
> - $b > 1$ gives **growth**; $0 < b < 1$ gives **decay**.
>
> ■ **Example**
>
> $f(x) = 3 \cdot 2^x$: $f(0) = 3$, $f(2) = 3 \cdot 4 = 12$; growth since $b = 2 > 1$.

2 Practice

2.1 In $f(x) = a \cdot b^x$, state what a represents. [1]

2.2 State the condition on b for growth. [1]

2.3 For $f(x) = 3 \cdot 2^x$, find $f(0)$ and $f(2)$. [2]

3 Exam-style questions

3.1 In $f(x) = a \cdot b^x$, the value a is the [1]

- **A** base
- **B** initial value
- **C** exponent
- **D** asymptote

3.2 The function $f(x) = 5 \cdot (0.5)^x$ represents [1]

- **A** growth
- **B** decay
- **C** linear change
- **D** a constant

3.3 $f(x) = 4 \cdot 3^x$.

(a) State $f(0)$. [1]

(b) State $f(1)$. [1]

(c) Is it growth or decay? [1]

Solutions

2.1 the initial value (the output at $x = 0$).

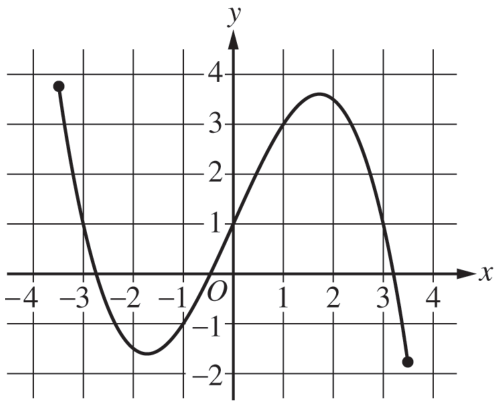
2.2 $b > 1$.

2.3 $f(0) = 3$; $f(2) = 3 \cdot 2^2 = 12$.

3.1 **B**.

3.2 **B**.

3.3 (a) 4. (b) 12. (c) growth.



Graph of f

1. The figure shows the graph of the function f on its domain of $-3.5 \le x \le 3.5$. The points $(-3, 1)$, $(0, 1)$, and $(3, 1)$ are on the graph of f. The function g is given by $g(x) = 2.916 \cdot (0.7)^x$.

   (A) (i) The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(3)$ as a decimal approximation, or indicate that it is not defined.

   (ii) Find all values of x for which $f(x) = 1$, or indicate that there are no such values.

   (B) (i) Find all values of x, as decimal approximations, for which $g(x) = 2$, or indicate that there are no such values.

   (ii) Determine the end behavior of g as x increases without bound. Express your answer using the mathematical notation of a limit.

   (C) (i) Determine if f has an inverse function.

   (ii) Give a reason for your answer based on the definition of a function and the graph of $y = f(x)$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

2.4 Exponential Function Manipulation

Name: ______ Class: ______ Date: ______ **Total: 9 marks**

KEY WORDS laws of exponents 指数律 • exponential function 指数函数

Objective

Build the skills to answer exam questions on **exponential function manipulation**.

You must be able to:

- apply the **laws of exponents 指数律** (product, quotient, power)
- rewrite $a \cdot b^{x+c}$ as a constant times b^x

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ **Laws of exponents**

$b^{m+n} = b^m \cdot b^n$; $b^{m-n} = \dfrac{b^m}{b^n}$; $(b^m)^n = b^{mn}$.

■ **Rewriting a shift as a scaling**

$a \cdot b^{x+c} = (a \cdot b^c) \cdot b^x$ — a horizontal shift of an exponential is the same as a vertical scaling.

■ **Example**

$5 \cdot 2^{x+2} = 5 \cdot 2^2 \cdot 2^x = 20 \cdot 2^x$.

2 Practice

2.1 Simplify $b^3 \cdot b^4$. [1]

2.2 Simplify $(b^2)^5$. [1]

2.3 Rewrite 2^{x+3} as a constant times 2^x. [2]

3 Exam-style questions

3.1 $b^m \cdot b^n$ equals [1]

- **A** b^{mn}
- **B** b^{m+n}
- **C** b^{m-n}
- **D** $b^m + b^n$

3.2 $(b^m)^n$ equals [1]

- **A** b^{m+n}
- **B** b^{mn}
- **C** $b^m \cdot n$
- **D** b^{m-n}

3.3 Rewrite $5 \cdot 2^{x+2}$ as a constant times 2^x.

(a) Split the exponent. [1]

(b) Evaluate 2^2. [1]

(c) Write the result as a constant times 2^x. [1]

Solutions

2.1 b^7.

2.2 b^{10}.

2.3 $2^{x+3} = 2^3 \cdot 2^x = 8 \cdot 2^x$.

3.1 **B**.

3.2 **B**.

3.3 (a) $5 \cdot 2^{x+2} = 5 \cdot 2^2 \cdot 2^x$. (b) $2^2 = 4$. (c) $20 \cdot 2^x$.

4. Directions:

- Unless otherwise specified, the domain of a function *f* is assumed to be the set of all real numbers *x* for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

A. The functions *g* and *h* are given by

$g(x) = e^{2x}$
$h(x) = \log_2(5x)$

i. Solve $g(x) = \frac{1}{e^6}$ for values of *x* in the domain of *g*.

ii. Solve $h(x) = 3$ for values of *x* in the domain of *h*.

B. The functions *j* and *k* are given by

$j(x) = 7^{(3x+1)} \cdot 7^x$
$k(x) = \sin(2x)\sec x$

i. Rewrite $j(x)$ as an expression of the form $7^{(\text{expression})}$.

ii. Rewrite $k(x)$ as an expression in which $\sin x$ appears exactly once and no other trigonometric functions are involved.

C. The function *m* is given by $m(x) = \tan^2(3x)$.

Find all input values for *m* in the interval $\left[0, \frac{\pi}{2}\right]$ that yield an output value of 1.

STOP

END OF EXAM

2.5 Exponential Function Context and Data Modeling

Name: ____________ Class: ________ Date: __________ **Total: 9 marks**

KEY WORDS growth/decay factor 增长 • percentage 百分比 • exponential function 指数函数 • initial value 初始值

Objective

Build the skills to answer exam questions on **exponential function context and data modeling**.

You must be able to:

- turn a **percentage 百分比** growth or decay rate into a **growth/decay factor 增长/衰减因子** b
- write an exponential model $f(x) = a \cdot b^x$ from a context

1 Worked examples

Study these first. Each one shows the method for a question type used later.

Growth and decay factors

A quantity starting at a and changing by $p\%$ per period has factor

$$b = 1 + \frac{p}{100} \text{ (growth)} \quad \text{or} \quad b = 1 - \frac{p}{100} \text{ (decay)}.$$

Example

\$500 growing 4% per year: $f(x) = 500 \cdot (1.04)^x$.

2 Practice

2.1 State the growth factor for 5% growth per year. [1]

2.2 State the decay factor for 20% decay per year. [1]

2.3 Write a model for $200 growing 10% per year. [2]

3 Exam-style questions

3.1 A quantity grows 3% per year. The growth factor b is [1]

- **A** 0.03
- **B** 0.97
- **C** 1.03
- **D** 3

3.2 A quantity decays 15% per year. The factor b is [1]

- **A** 1.15
- **B** 0.85
- **C** 0.15
- **D** -0.15

3.3 A colony of 100 bacteria doubles each hour.

(a) State the initial value. [1]

(b) State the growth factor. [1]

(c) Write the model $f(t)$. [1]

Solutions

2.1 1.05.

2.2 0.80.

2.3 $f(x) = 200 \cdot (1.10)^x$.

3.1 **C**.

3.2 **B**.

3.3 (a) 100. (b) 2. (c) $f(t) = 100 \cdot 2^t$.

2. A person purchased a car at the end of the year 2019 ($t = 0$). The car's value decreases over time. At the end of the year 2020 ($t = 1$), the car was valued at 27.2 thousand dollars. At the end of the year 2025 ($t = 6$), the car was valued at 14.8 thousand dollars.

The value of the car can be modeled by the function V given by $V(t) = ab^t$, where $V(t)$ is the value of the car, in thousands of dollars, at time t, and t is the number of years since the end of 2019.

A.

i. Use the given data to write two equations that can be used to find the values for constants a and b in the expression for $V(t)$.

ii. Find the values for a and b as decimal approximations.

B.

i. Use the given data to find the average rate of change of the value of the car, in thousands of dollars per year, from $t = 1$ to $t = 6$ years. Express your answer as a decimal approximation. Show the computations that lead to your answer.

ii. Use the average rate of change found in part B (i) to estimate the value of the car, in thousands of dollars, at $t = 3$ years. Show the work that leads to your answer.

iii. The average rate of change found in part B (i) can be used to determine the secant line for the graph of V on the interval $1 \le t \le 6$. Let $A(t)$ represent the estimate of the value of the car, in thousands of dollars, at time t years, using the secant line. For $A(3)$, found in part B (ii), it can be shown that $A(3) > V(3)$.

In general, $A(t) > V(t)$ for all t, where $1 < t < 6$. Use the secant line and the graph of V to explain why this is true.

C. When the value of the car reaches 2 thousand dollars, the car's owner plans to donate it to an auto mechanic school. As a result, the car will immediately lose all of its value. Explain how this information can be used to determine a domain limitation for the model V.

END OF PART A

2.6 Competing Function Model Validation

Name: ______________ Class: __________ Date: ____________ **Total: 8 marks**

KEY WORDS residual 残差 • residual plot 残差图

Objective

Build the skills to answer exam questions on **competing function model validation**.

You must be able to:

- compute a **residual** 残差 (observed minus predicted)
- use residuals to compare and validate competing models

1 Worked examples

Study these first. Each one shows the method for a question type used later.

> ■ **Residuals**
>
> $$\text{residual} = \text{observed} - \text{predicted}.$$
>
> A model fits well when the residuals are **small** and show **no pattern**. To choose between two candidate models, compare their residuals.
>
> ■ **Example**
>
> Observed 20, predicted 18 $\rightarrow$ residual $= 2$ (the model under-predicted).

2 Practice

2.1 Define a residual. [1]

2.2 State what small, patternless residuals indicate. [1]

2.3 The observed value is 20 and the predicted value is 18. Find the residual. [1]

3 Exam-style questions

3.1 A residual is [1]

- **A** predicted minus observed
- **B** observed minus predicted
- **C** observed times predicted
- **D** the slope

3.2 A well-fitting model has residuals that are [1]

- **A** large and patterned
- **B** small and patternless
- **C** all positive
- **D** steadily increasing

3.3 A model predicts 50; the actual value is 46.

(a) Find the residual. [1]

(b) State whether the model over- or under-predicted. [1]

(c) State what a residual plot should look like for a good fit. [1]

Solutions

2.1 the observed value minus the predicted value.

2.2 the model fits the data well.

2.3 $20 - 18 = 2$.

3.1 **B**.

3.2 **B**.

3.3 (a) $46 - 50 = -4$. (b) over-predicted. (c) scattered around zero with no pattern.

1. The function f is decreasing and is defined for all real numbers. The table gives values for $f(x)$ at selected values of x.

x	-2	-1	0	1	2
$f(x)$	14	7	3.5	1.75	0.875

The function g is given by $g(x) = -0.167x^3 + x^2 - 1.834$.

A.

i. The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(1)$ as a decimal approximation, or indicate that it is not defined. Show the work that leads to your answer.

ii. Find the value of $f^{-1}(3.5)$, or indicate that it is not defined.

B.

i. Find all values of x, as decimal approximations, for which $g(x) = 0$, or indicate that there are no such values.

ii. Determine the end behavior of g as x increases without bound. Express your answer using the mathematical notation of a limit.

C.

i. Based on the table, which of the following function types best models function f: linear, quadratic, exponential, or logarithmic?

ii. Give a reason for your answer in part C (i) based on the relationship between the change in the output values of f and the change in the input values of f. Refer to the values in the table in your reasoning.

2.7 Composition of Functions

Name: ______________ Class: __________ Date: ___________ **Total: 9 marks**

KEY WORDS composite function 复合函数

Objective

Build the skills to answer exam questions on **composition of functions**.

You must be able to:

- evaluate a **composite function** 复合函数 $(f \circ g)(x) = f(g(x))$
- apply the inner function first, then the outer

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ **Composite functions**

$(f \circ g)(x) = f(g(x))$ — apply g **first**, then feed the result into f.

■ **Example**

$f(x) = x + 1$, $g(x) = x^2$:

$(f \circ g)(x) = x^2 + 1$, while $(g \circ f)(x) = (x + 1)^2$ — order matters.

2 Practice

2.1 State what $(f \circ g)(x)$ means. [1]

2.2 For $f(x) = x + 2$ and $g(x) = 3x$, find $(f \circ g)(x)$. [2]

2.3 For the same functions, find $(g \circ f)(x)$. [1]

3 Exam-style questions

3.1 $(f \circ g)(x)$ means [1]

- **A** $f(x) \cdot g(x)$
- **B** $f(g(x))$
- **C** $g(f(x))$
- **D** $f(x) + g(x)$

3.2 In a composition, you apply the [1]

- **A** outer function first
- **B** inner function first
- **C** both at once
- **D** neither

3.3 $f(x) = 2x$, $g(x) = x - 1$.

(a) Find $(f \circ g)(x)$. [1]

(b) Find $(g \circ f)(x)$. [1]

(c) Evaluate $(f \circ g)(4)$. [1]

Solutions

2.1 apply g first, then f: $f(g(x))$.

2.2 $(f \circ g)(x) = 3x + 2$.

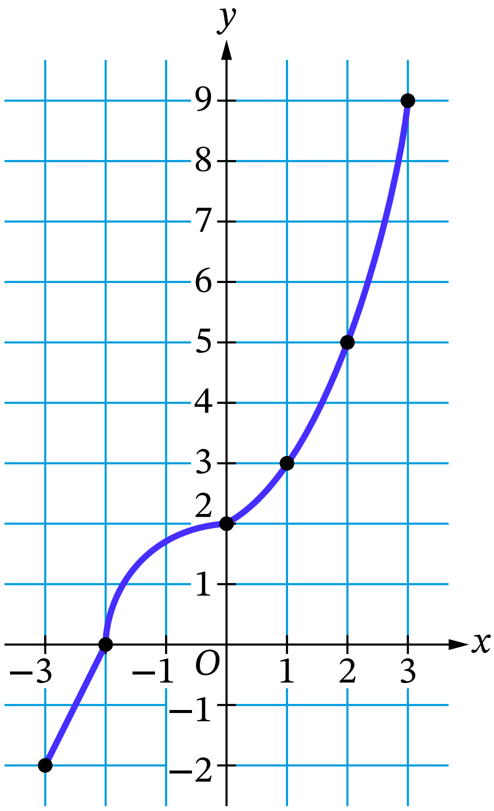
2.3 $(g \circ f)(x) = (x + 2) \cdot 3 = 3x + 6$.

3.1 **B**.

3.2 **B**.

3.3 (a) $2(x - 1) = 2x - 2$. (b) $2x - 1$. (c) $2(4 - 1) = 6$.

1. The figure shows the graph of the increasing function f on its domain of $-3 \le x \le 3$.



Graph of f

The function g is given by $g(x) = -4.792 + \ln(6x - 6)$.

A.

i. The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(2)$ as a decimal approximation, or indicate that it is not defined. Show the work that leads to your answer.

ii. Find all values of x for which $f(x) = 3$, or indicate that there are no such values.

B.

i. Find all values of x, as decimal approximations, for which $g(x) = -1.5$, or indicate that there are no such values.

ii. Determine the behavior of g as x-values decrease and get arbitrarily close to 1. Express your answer using the mathematical notation of a limit.

C.

i. Is the function f invertible?

ii. Give a reason for your answer in part C (i) based on properties of the function f. Refer to points on the graph of f in your reasoning.

1. The function *f* is decreasing and is defined for all real numbers. The table gives values for $f(x)$ at selected values of *x*.

x	-2	-1	0	1	2
$f(x)$	14	7	3.5	1.75	0.875

The function *g* is given by $g(x) = -0.167x^3 + x^2 - 1.834$.

A.

i. The function *h* is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(1)$ as a decimal approximation, or indicate that it is not defined. Show the work that leads to your answer.

ii. Find the value of $f^{-1}(3.5)$, or indicate that it is not defined.

B.

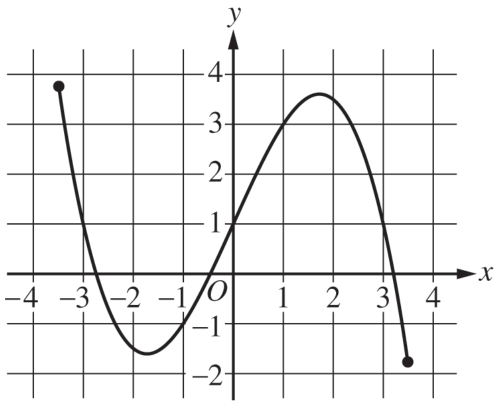
i. Find all values of *x*, as decimal approximations, for which $g(x) = 0$, or indicate that there are no such values.

ii. Determine the end behavior of *g* as *x* increases without bound. Express your answer using the mathematical notation of a limit.

C.

i. Based on the table, which of the following function types best models function *f*: linear, quadratic, exponential, or logarithmic?

ii. Give a reason for your answer in part C (i) based on the relationship between the change in the output values of *f* and the change in the input values of *f*. Refer to the values in the table in your reasoning.



Graph of f

1. The figure shows the graph of the function f on its domain of $-3.5 \le x \le 3.5$. The points $(-3, 1)$, $(0, 1)$, and $(3, 1)$ are on the graph of f. The function g is given by $g(x) = 2.916 \cdot (0.7)^x$.

   (A) (i) The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(3)$ as a decimal approximation, or indicate that it is not defined.

   (ii) Find all values of x for which $f(x) = 1$, or indicate that there are no such values.

   (B) (i) Find all values of x, as decimal approximations, for which $g(x) = 2$, or indicate that there are no such values.

   (ii) Determine the end behavior of g as x increases without bound. Express your answer using the mathematical notation of a limit.

   (C) (i) Determine if f has an inverse function.

   (ii) Give a reason for your answer based on the definition of a function and the graph of $y = f(x)$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

2.8 Inverse Functions

Name: ____________ Class: ________ Date: __________ **Total: 9 marks**

KEY WORDS inverse function 反函数

Objective

Build the skills to answer exam questions on **inverse functions**.

You must be able to:

- find an **inverse function 反函数** by swapping x and y and solving
- use that the graph of f^{-1} reflects f across the line $y = x$

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Inverses

f^{-1} **undoes** f: $(f \circ f^{-1})(x) = x$. Find it by swapping x and y and solving for y. The graph reflects across $y = x$. Only **one-to-one** functions have an inverse function.

■ Example

$f(x) = 2x + 1$: swap $\to x = 2y + 1 \to y = \dfrac{x-1}{2}$, so $f^{-1}(x) = \dfrac{x-1}{2}$.

2 Practice

2.1 State what an inverse function does. [1]

2.2 State the line of reflection between f and f^{-1}. [1]

2.3 Find the inverse of $f(x) = x + 5$. [2]

3 Exam-style questions

3.1 The graph of f^{-1} is the reflection of f across [1]

- **A** the x-axis
- **B** the y-axis
- **C** the line $y = x$
- **D** the origin

3.2 A function has an inverse function only if it is [1]

- **A** increasing
- **B** one-to-one
- **C** linear
- **D** positive

3.3 $f(x) = 3x - 6$.

(a) Swap x and y. [1]

(b) Solve for y. [1]

(c) State $f^{-1}(x)$. [1]

Solutions

2.1 it undoes f, returning the original input.

2.2 the line $y = x$.

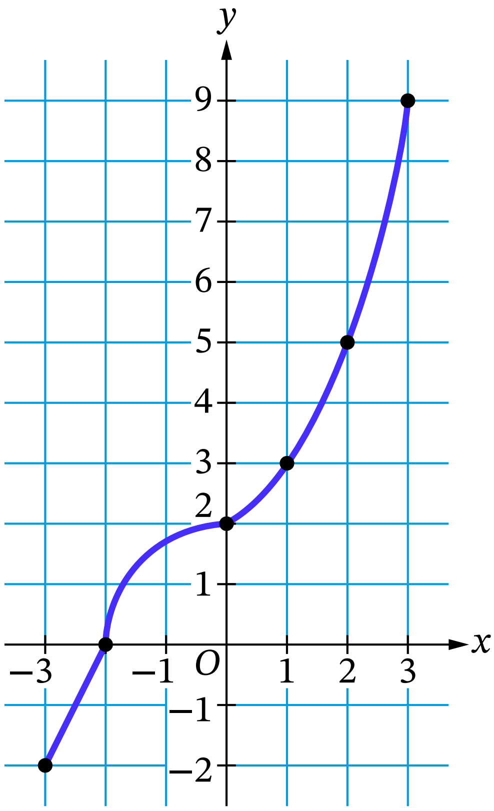
2.3 swap: $x = y + 5$; solve: $y = x - 5$, so $f^{-1}(x) = x - 5$.

3.1 **C**.

3.2 **B**.

3.3 (a) $x = 3y - 6$. (b) $y = \dfrac{x+6}{3}$. (c) $f^{-1}(x) = \dfrac{x+6}{3}$.

1. The figure shows the graph of the increasing function f on its domain of $-3 \le x \le 3$.



Graph of f

The function g is given by $g(x) = -4.792 + \ln(6x - 6)$.

A.

i. The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(2)$ as a decimal approximation, or indicate that it is not defined. Show the work that leads to your answer.

ii. Find all values of x for which $f(x) = 3$, or indicate that there are no such values.

B.

i. Find all values of x, as decimal approximations, for which $g(x) = -1.5$, or indicate that there are no such values.

ii. Determine the behavior of g as x-values decrease and get arbitrarily close to 1. Express your answer using the mathematical notation of a limit.

C.

i. Is the function f invertible?

ii. Give a reason for your answer in part C (i) based on properties of the function f. Refer to points on the graph of f in your reasoning.

1. The function f is decreasing and is defined for all real numbers. The table gives values for $f(x)$ at selected values of x.

x	-2	-1	0	1	2
$f(x)$	14	7	3.5	1.75	0.875

The function g is given by $g(x) = -0.167x^3 + x^2 - 1.834$.

A.

i. The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(1)$ as a decimal approximation, or indicate that it is not defined. Show the work that leads to your answer.

ii. Find the value of $f^{-1}(3.5)$, or indicate that it is not defined.

B.

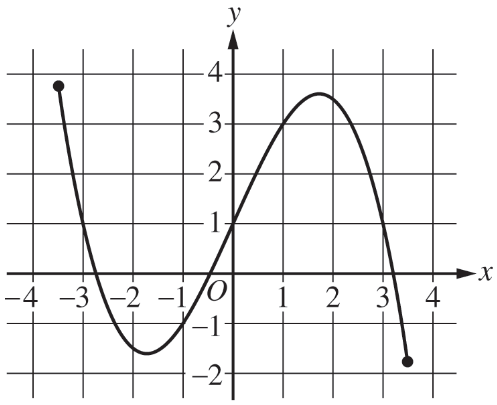
i. Find all values of x, as decimal approximations, for which $g(x) = 0$, or indicate that there are no such values.

ii. Determine the end behavior of g as x increases without bound. Express your answer using the mathematical notation of a limit.

C.

i. Based on the table, which of the following function types best models function f: linear, quadratic, exponential, or logarithmic?

ii. Give a reason for your answer in part C (i) based on the relationship between the change in the output values of f and the change in the input values of f. Refer to the values in the table in your reasoning.



Graph of f

1. The figure shows the graph of the function f on its domain of $-3.5 \leq x \leq 3.5$. The points $(-3, 1)$, $(0, 1)$, and $(3, 1)$ are on the graph of f. The function g is given by $g(x) = 2.916 \cdot (0.7)^x$.

   (A) (i) The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(3)$ as a decimal approximation, or indicate that it is not defined.

   (ii) Find all values of x for which $f(x) = 1$, or indicate that there are no such values.

   (B) (i) Find all values of x, as decimal approximations, for which $g(x) = 2$, or indicate that there are no such values.

   (ii) Determine the end behavior of g as x increases without bound. Express your answer using the mathematical notation of a limit.

   (C) (i) Determine if f has an inverse function.

   (ii) Give a reason for your answer based on the definition of a function and the graph of $y = f(x)$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

2.9 Logarithmic Expressions

Name: ______________ Class: __________ Date: ____________ **Total: 9 marks**

KEY WORDS logarithmic 对数 • base 底数

Objective

Build the skills to answer exam questions on **logarithmic expressions**.

You must be able to:

- convert between **logarithmic** 对数 and exponential form
- evaluate simple logarithms

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Logarithms

$\log_b(y) = x$ means exactly $b^x = y$. A logarithm answers the question "**what exponent?**".

- $\log_{10}$ is the **common log**;
- ln is the **natural log** (base e).

■ Example

$\log_2(8) = 3$ because $2^3 = 8$.

2 Practice

2.1 Rewrite $\log_b(y) = x$ in exponential form. [1]

2.2 Evaluate $\log_2(8)$. [1]

2.3 Evaluate $\log_{10}(1000)$. [2]

3 Exam-style questions

3.1 $\log_b(y) = x$ is equivalent to [1]

- **A** $b^y = x$
- **B** $b^x = y$
- **C** $x^b = y$
- **D** $y^x = b$

3.2 $\log_3(9)$ equals [1]

- **A** 2
- **B** 3
- **C** 9
- **D** 27

3.3 Evaluate each.

(a) $\log_5(25)$. [1]

(b) $\log_{10}(100)$. [1]

(c) $\log_2(1)$. [1]

Solutions

2.1 $b^x = y$.

2.2 3.

2.3 $\log_{10}(1000) = 3$ since $10^3 = 1000$.

3.1 **B**.

3.2 **A**.

3.3 (a) 2. (b) 2. (c) 0.

2.10 Inverses of Exponential Functions

Name: ______ Class: ______ Date: ______ **Total: 9 marks**

KEY WORDS logarithm 对数 • exponential function 指数函数 • root 方根

Objective

Build the skills to answer exam questions on **inverses of exponential functions**.

You must be able to:

- use that the inverse of b^x is $\log_b(x)$
- solve an exponential equation by taking a logarithm

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ **Log inverts exponent**

The inverse of $f(x) = b^x$ is $f^{-1}(x) = \log_b(x)$. To solve $b^x = y$, take $\log_b$ of both sides: $x = \log_b(y)$.

■ **Example**

$2^x = 16 \;\Rightarrow\; x = \log_2(16) = 4$.

2 Practice

2.1 State the inverse of $f(x) = 10^x$. [1]

2.2 Solve $3^x = 81$. [2]

2.3 State the inverse of e^x. [1]

3 Exam-style questions

3.1 The inverse of $f(x) = b^x$ is [1]

- **A** b^{-x}
- **B** $\log_b(x)$
- **C** $1/b^x$
- **D** x^b

3.2 To solve $5^x = 125$, take [1]

- **A** $\log_5$ of both sides
- **B** the square root
- **C** the reciprocal
- **D** the derivative

3.3 Solve $2^x = 32$.

(a) Rewrite 32 as a power of 2. [1]

(b) Equate the exponents. [1]

(c) State x. [1]

Solutions

2.1 $f^{-1}(x) = \log_{10}(x)$.

2.2 $3^x = 81 = 3^4$, so $x = 4$.

2.3 $\ln(x)$ (i.e. $\log_e(x)$).

3.1 **B**.

3.2 **A**.

3.3 (a) $32 = 2^5$. (b) $x = 5$. (c) 5.

4. **Directions:**

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

(A) The functions g and h are given by

$$g(x) = e^{(x+3)}$$
$$h(x) = \arcsin\left(\frac{x}{2}\right).$$

(i) Solve $g(x) = 10$ for values of x in the domain of g.

(ii) Solve $h(x) = \frac{\pi}{4}$ for values of x in the domain of h.

(B) The functions j and k are given by

$$j(x) = \log_{10}(8x^5) + \log_{10}(2x^2) - 9\log_{10}x$$
$$k(x) = \left(\frac{1 - \sin^2 x}{\sin x}\right)\sec x.$$

(i) Rewrite $j(x)$ as a single logarithm base 10 without negative exponents in any part of the expression. Your result should be of the form $\log_{10}(\text{expression})$.

(ii) Rewrite $k(x)$ as a single term involving $\tan x$.

(C) The function m is given by

$$m(x) = \cos^{-1}(\tan(2x)).$$

Find all values in the domain of m that yield an output value of 0.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

STOP

END OF EXAM

2.11 Logarithmic Functions

Name: ______________ Class: _________ Date: ____________ **Total: 9 marks**

KEY WORDS domain 定义域 • vertical asymptote 垂直渐近线 • logarithmic function 对数函数 • logarithm 对数

Objective

Build the skills to answer exam questions on **logarithmic functions**.

You must be able to:

- state the **domain 定义域**, **vertical asymptote 垂直渐近线**, and key point of $f(x) = \log_b(x)$
- relate the log graph to the exponential graph

1 Worked examples

Study these first. Each one shows the method for a question type used later.

The logarithmic function

$f(x) = \log_b(x)$ has:

- domain $x > 0$, range all real numbers;
- a **vertical asymptote** at $x = 0$;
- it always passes through $(1, 0)$;
- for $b > 1$ it is increasing.

Its graph is the reflection of b^x across $y = x$.

Reading a log graph

For $f(x) = \log_2(x)$: $f(1) = 0$, $f(2) = 1$, $f(4) = 2$, $f(8) = 3$ – each time x **doubles**, f rises by 1. The curve climbs without bound but never reaches the line $x = 0$ (its vertical asymptote), and $f(x)$ is undefined for $x \leq 0$.

2 Practice

2.1 State the domain of $f(x) = \log_b(x)$. [1]

2.2 State the vertical asymptote of the log function. [1]

2.3 State the point every $\log_b$ graph passes through. [2]

3 Exam-style questions

3.1 The domain of $f(x) = \log_b(x)$ is [1]

- **A** all real numbers
- **B** $x > 0$
- **C** $x \geq 0$
- **D** $x \neq 0$

3.2 The graph of $\log_b(x)$ has a vertical asymptote at [1]

- **A** $x = 0$
- **B** $x = 1$
- **C** $y = 0$
- **D** $y = 1$

3.3 $f(x) = \log_2(x)$.

(a) State $f(1)$. [1]

(b) State $f(2)$. [1]

(c) State $f(4)$. [1]

Solutions

2.1 $x > 0$.

2.2 $x = 0$.

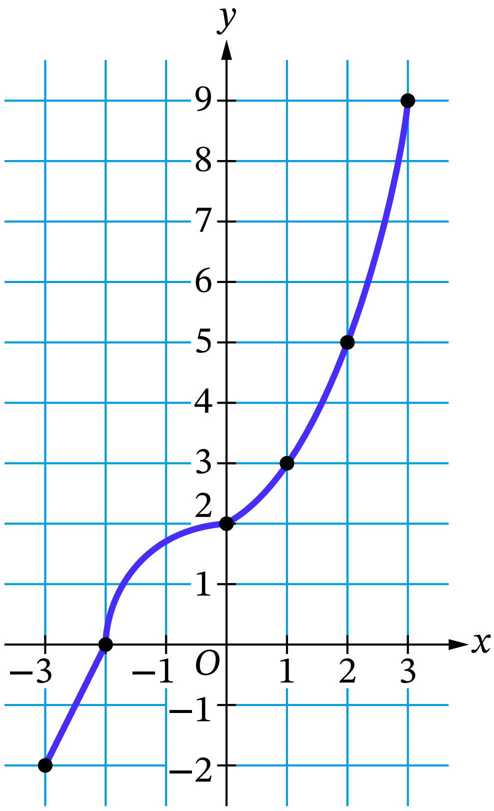
2.3 $(1, 0)$.

3.1 **B**.

3.2 **A**.

3.3 (a) 0. (b) 1. (c) 2.

1. The figure shows the graph of the increasing function f on its domain of $-3 \le x \le 3$.



Graph of f

The function g is given by $g(x) = -4.792 + \ln(6x - 6)$.

A.

i. The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(2)$ as a decimal approximation, or indicate that it is not defined. Show the work that leads to your answer.

ii. Find all values of x for which $f(x) = 3$, or indicate that there are no such values.

B.

i. Find all values of x, as decimal approximations, for which $g(x) = -1.5$, or indicate that there are no such values.

ii. Determine the behavior of g as x-values decrease and get arbitrarily close to 1. Express your answer using the mathematical notation of a limit.

C.

i. Is the function f invertible?

ii. Give a reason for your answer in part C (i) based on properties of the function f. Refer to points on the graph of f in your reasoning.

2.12 Logarithmic Function Manipulation

Name: ____________ Class: ________ Date: __________ **Total: 9 marks**

KEY WORDS laws of logarithms 对数律 • change-of-base 换底 • logarithm 对数 • base 底数 • logarithmic function

Objective

Build the skills to answer exam questions on **logarithmic function manipulation**.

You must be able to:

- apply the **laws of logarithms** 对数律 (product, quotient, power)
- use the **change-of-base** 换底 formula

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ **Laws of logarithms**

$$\log_b(xy) = \log_b x + \log_b y, \quad \log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y, \quad \log_b(x^n) = n\log_b x.$$

■ **Change of base**

$$\log_b(x) = \frac{\ln x}{\ln b}.$$

■ **Example**

$\log_2(8x) = \log_2 8 + \log_2 x = 3 + \log_2 x.$

2 Practice

2.1 Expand $\log(xy)$. [1]

2.2 Write $\log(x^3)$ using the power rule. [1]

2.3 Combine $\log 6 + \log 2$ into a single logarithm. [2]

3 Exam-style questions

3.1 $\log_b(xy)$ equals [1]

- **A** $\log_b x \cdot \log_b y$
- **B** $\log_b x + \log_b y$
- **C** $\log_b x - \log_b y$
- **D** $\log_b(x + y)$

3.2 $\log_b(x^n)$ equals [1]

- **A** $n + \log_b x$
- **B** $n \log_b x$
- **C** $(\log_b x)^n$
- **D** $\log_b(nx)$

3.3 Expand $\log_2(8x)$.

(a) Apply the product rule. [1]

(b) Evaluate $\log_2 8$. [1]

(c) Write the final expression. [1]

Solutions

2.1 $\log x + \log y$.

2.2 $3 \log x$.

2.3 $\log(6 \cdot 2) = \log 12$.

3.1 **B**.

3.2 **B**.

3.3 (a) $\log_2 8 + \log_2 x$. (b) 3. (c) $3 + \log_2 x$.

4. Directions:

- Unless otherwise specified, the domain of a function *f* is assumed to be the set of all real numbers *x* for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x+3x$, $5^2\cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

A. The functions *g* and *h* are given by

$$g(x) = 2\log_3 x$$
$$h(x) = 4\cos^2 x$$

i. Solve $g(x) = 4$ for values of *x* in the domain of *g*.

ii. Solve $h(x) = 3$ for values of *x* in the interval $\left[0, \frac{\pi}{2}\right)$.

B. The functions *j* and *k* are given by

$$j(x) = \log_2 x + 3\log_2 2$$
$$k(x) = \frac{6}{\tan x(\csc^2 x - 1)}$$

i. Rewrite $j(x)$ as a single logarithm base 2 without negative exponents in any part of the expression. Your result should be of the form $\log_2(\text{expression})$.

ii. Rewrite $k(x)$ as an expression in which $\tan x$ appears exactly once and no other trigonometric functions are involved.

C. The function *m* is given by $m(x) = e^{2x} - e^{x} - 12$. Find all input values in the domain of *m* that yield an output value of 0.

STOP

END OF EXAM

4. **Directions:**

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

(A) The functions g and h are given by

$$g(x) = e^{(x+3)}$$
$$h(x) = \arcsin\left(\frac{x}{2}\right).$$

(i) Solve $g(x) = 10$ for values of x in the domain of g.

(ii) Solve $h(x) = \frac{\pi}{4}$ for values of x in the domain of h.

(B) The functions j and k are given by

$$j(x) = \log_{10}(8x^5) + \log_{10}(2x^2) - 9\log_{10}x$$
$$k(x) = \left(\frac{1 - \sin^2 x}{\sin x}\right)\sec x.$$

(i) Rewrite $j(x)$ as a single logarithm base 10 without negative exponents in any part of the expression. Your result should be of the form $\log_{10}(\text{expression})$.

(ii) Rewrite $k(x)$ as a single term involving $\tan x$.

(C) The function m is given by

$$m(x) = \cos^{-1}(\tan(2x)).$$

Find all values in the domain of m that yield an output value of 0.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

STOP

END OF EXAM

2.13 Exponential and Logarithmic Equations and Inequalities

Name: ____________ Class: ________ Date: __________ **Total: 9 marks**

KEY WORDS domain 定义域 • logarithm 对数 • base 底数 • root 方根

Objective

Build the skills to answer exam questions on **exponential and logarithmic equations and inequalities**.

You must be able to:

- solve $b^x = c$ by taking a **logarithm** 对数
- solve $\log_b(x) = c$ by rewriting in exponential form, and check the **domain** 定义域

1 Worked examples

Study these first. Each one shows the method for a question type used later.

Solving

- $b^x = c \;\Rightarrow\; x = \log_b c$;
- $\log_b(x) = c \;\Rightarrow\; x = b^c$.

Always check a log solution lies in the domain (the argument must be > 0).

Example

$\log_2(x) = 5 \;\Rightarrow\; x = 2^5 = 32$ (and $32 > 0$, so it is valid).

2 Practice

2.1 Solve $10^x = 1000$. [1]

2.2 Solve $\log_2(x) = 5$. [2]

2.3 State the domain restriction to check for a log equation. [1]

3 Exam-style questions

3.1 To solve $2^x = 20$, you take [1]

- **A** log base 2 of both sides
- **B** the square root
- **C** the reciprocal
- **D** nothing

3.2 The solution of $\log_3(x) = 2$ is [1]

- **A** 6
- **B** 8
- **C** 9
- **D** 5

3.3 Solve $\log_5(x) = 3$.

(a) Rewrite in exponential form. [1]

(b) Evaluate. [1]

(c) Confirm the domain is satisfied. [1]

Solutions

2.1 $x = 3$.

2.2 $x = 2^5 = 32$.

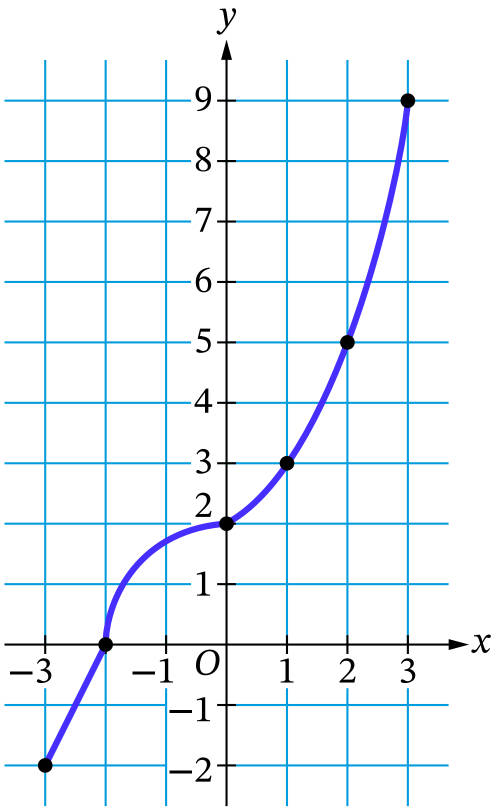
2.3 the argument of the log must be positive ($x > 0$).

3.1 **A**.

3.2 **C**.

3.3 (a) $x = 5^3$. (b) 125. (c) $125 > 0$, so it is valid.

1. The figure shows the graph of the increasing function f on its domain of $-3 \le x \le 3$.



Graph of f

The function g is given by $g(x) = -4.792 + \ln(6x - 6)$.

A.

i. The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(2)$ as a decimal approximation, or indicate that it is not defined. Show the work that leads to your answer.

ii. Find all values of x for which $f(x) = 3$, or indicate that there are no such values.

B.

i. Find all values of x, as decimal approximations, for which $g(x) = -1.5$, or indicate that there are no such values.

ii. Determine the behavior of g as x-values decrease and get arbitrarily close to 1. Express your answer using the mathematical notation of a limit.

C.

i. Is the function f invertible?

ii. Give a reason for your answer in part C (i) based on properties of the function f. Refer to points on the graph of f in your reasoning.

4. Directions:

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

A. The functions g and h are given by

$$g(x) = e^{2x}$$
$$h(x) = \log_2(5x)$$

i. Solve $g(x) = \frac{1}{e^6}$ for values of x in the domain of g.

ii. Solve $h(x) = 3$ for values of x in the domain of h.

B. The functions j and k are given by

$$j(x) = 7^{(3x+1)} \cdot 7^x$$
$$k(x) = \sin(2x) \sec x$$

i. Rewrite $j(x)$ as an expression of the form $7^{(\text{expression})}$.

ii. Rewrite $k(x)$ as an expression in which $\sin x$ appears exactly once and no other trigonometric functions are involved.

C. The function m is given by $m(x) = \tan^2(3x)$.

Find all input values for m in the interval $\left[0, \frac{\pi}{2}\right]$ that yield an output value of 1.

STOP

END OF EXAM

4. Directions:

- Unless otherwise specified, the domain of a function *f* is assumed to be the set of all real numbers *x* for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x+3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

A. The functions *g* and *h* are given by

$$g(x) = 2\log_3 x$$
$$h(x) = 4\cos^2 x$$

i. Solve $g(x) = 4$ for values of *x* in the domain of *g*.

ii. Solve $h(x) = 3$ for values of *x* in the interval $\left[0, \frac{\pi}{2}\right)$.

B. The functions *j* and *k* are given by

$$j(x) = \log_2 x + 3\log_2 2$$
$$k(x) = \frac{6}{\tan x(\csc^2 x - 1)}$$

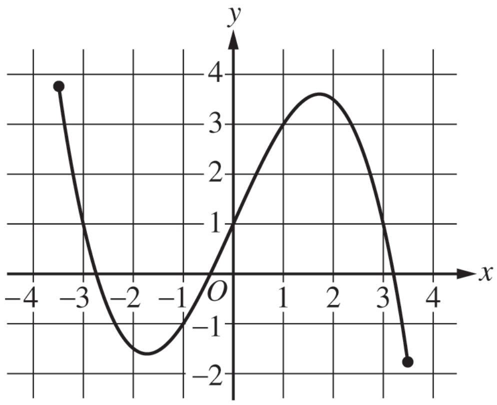
i. Rewrite $j(x)$ as a single logarithm base 2 without negative exponents in any part of the expression. Your result should be of the form $\log_2(\text{expression})$.

ii. Rewrite $k(x)$ as an expression in which $\tan x$ appears exactly once and no other trigonometric functions are involved.

C. The function *m* is given by $m(x) = e^{2x} - e^x - 12$. Find all input values in the domain of *m* that yield an output value of 0.

STOP

END OF EXAM



Graph of f

1. The figure shows the graph of the function f on its domain of $-3.5 \le x \le 3.5$. The points $(-3, 1)$, $(0, 1)$, and $(3, 1)$ are on the graph of f. The function g is given by $g(x) = 2.916 \cdot (0.7)^x$.

   (A) (i) The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(3)$ as a decimal approximation, or indicate that it is not defined.

   (ii) Find all values of x for which $f(x) = 1$, or indicate that there are no such values.

   (B) (i) Find all values of x, as decimal approximations, for which $g(x) = 2$, or indicate that there are no such values.

   (ii) Determine the end behavior of g as x increases without bound. Express your answer using the mathematical notation of a limit.

   (C) (i) Determine if f has an inverse function.

   (ii) Give a reason for your answer based on the definition of a function and the graph of $y = f(x)$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

2.14 Logarithmic Function Context and Data Modeling

Name: ______________ Class: __________ Date: ____________ **Total: 8 marks**

KEY WORDS logarithmic scale 对数刻度 • logarithm 对数 • base 底数 • logarithmic function 对数函数

Objective

Build the skills to answer exam questions on **logarithmic function context and data modeling**.

You must be able to:

- recognise real quantities measured on a **logarithmic scale** 对数刻度 (pH, decibels, Richter)
- interpret a change of one unit on a base-10 log scale

1 Worked examples

Study these first. Each one shows the method for a question type used later.

Logarithmic models and scales

A logarithmic model $y = a + b\ln(x)$ rises quickly, then levels off.

Logarithmic **scales** (pH, decibels, Richter) turn multiplicative steps into additive ones: on a base-10 scale, each +1 unit is a factor of 10.

Example

+2 units on a base-10 scale is a factor of $10^2 = 100$.

2 Practice

2.1 State one real quantity measured on a logarithmic scale. [1]

2.2 On a base-10 log scale, a change of 1 unit means what factor? [1]

2.3 State the shape of a logarithmic growth curve. [1]

3 Exam-style questions

3.1 The Richter scale for earthquakes is [1]

- **A** linear
- **B** logarithmic
- **C** exponential
- **D** quadratic

3.2 On a base-10 log scale, +2 units corresponds to a factor of [1]

- **A** 2
- **B** 10
- **C** 20
- **D** 100

3.3 A sound rises from 40 dB to 60 dB (a base-10 scale where +10 dB = ×10 intensity).

(a) State the increase in dB. [1]

(b) State how many factors of 10 this is. [1]

(c) State the intensity factor. [1]

Solutions

2.1 any of: pH, sound level in decibels, earthquake magnitude (Richter).

2.2 a factor of 10.

2.3 rises quickly at first, then levels off (increasing and concave down).

3.1 **B**.

3.2 **D**.

3.3 (a) 20 dB. (b) 2. (c) $\times 100$.

2. On the initial day of sales ($t = 0$) for a new video game, there were 40 thousand units of the game sold that day. Ninety-one days later ($t = 91$), there were 76 thousand units of the game sold that day.

   The number of units of the video game sold on a given day can be modeled by the function G given by $G(t) = a + b\ln(t + 1)$, where $G(t)$ is the number of units sold, in thousands, on day t since the initial day of sales.

   (A) (i) Use the given data to write two equations that can be used to find the values for constants a and b in the expression for $G(t)$.

   (ii) Find the values for a and b as decimal approximations.

   (B) (i) Use the given data to find the average rate of change of the number of units of the video game sold, in thousands per day, from $t = 0$ to $t = 91$ days. Express your answer as a decimal approximation. Show the computations that lead to your answer.

   (ii) Use the average rate of change found in (i) to estimate the number of units of the video game sold, in thousands, on day $t = 50$. Show the work that leads to your answer.

   (iii) Let A_t represent the estimate of the number of units of the video game sold, in thousands, using the average rate of change found in (i). For A_{50}, found in (ii), it can be shown that $A_{50} < G(50)$. Explain why, in general, $A_t < G(t)$ for all t, where $0 < t < 91$.

   (C) The makers of the video game reported that daily sales of the video game decreased each day after $t = 91$. Explain why the error in the model G increases after $t = 91$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

END OF PART A

PRECALCULUS

SECTION II, Part B

Time—30 minutes

2 Questions

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.

2.15 Semi-log Plots

Name: ______________ Class: __________ Date: ____________ **Total: 9 marks**

KEY WORDS semi-log plot 半对数图

Objective

Build the skills to answer exam questions on **semi-log plots**.

You must be able to:

- explain how a **semi-log plot 半对数图** linearizes exponential data
- relate the slope and intercept to the model $y = a \cdot b^x$

1 Worked examples

Study these first. Each one shows the method for a question type used later.

Semi-log plots

A semi-log plot graphs $\log(y)$ against x. For exponential data $y = a \cdot b^x$,

$$\log(y) = \log a + x \log b,$$

which is **linear** in x. So exponential data appear as a **straight line** on a semi-log plot; the slope is $\log b$ and the intercept is $\log a$.

From the line back to the model

Suppose a semi-log plot of some data is a straight line with slope 0.3 and intercept 1. Then $\log b = 0.3$ and $\log a = 1$, so $b = 10^{0.3} \approx 2$ and $a = 10^1 = 10$. The exponential model is $y \approx 10 \cdot 2^x$.

2 Practice

2.1 State what a semi-log plot graphs. [1]

2.2 State what a straight line on a semi-log plot indicates. [1]

2.3 For $y = a \cdot b^x$, what does $\log(y)$ equal? [2]

3 Exam-style questions

3.1 A semi-log plot graphs [1]

- **A** y vs x
- **B** $\log(y)$ vs x
- **C** $\log(y)$ vs $\log(x)$
- **D** x vs y

3.2 A straight line on a semi-log plot indicates the data are [1]

- **A** linear
- **B** exponential
- **C** quadratic
- **D** constant

3.3 $y = a \cdot b^x$.

(a) Take log of both sides. [1]

(b) State the slope of $\log(y)$ vs x. [1]

(c) State the vertical intercept. [1]

Solutions

2.1 $\log(y)$ against x.

2.2 the data are exponential.

2.3 $\log(y) = \log a + x \log b$.

3.1 **B**.

3.2 **B**.

3.3 (a) $\log(y) = \log a + x \log b$. (b) $\log b$. (c) $\log a$.