Week 1

AP Precalculus

Homework bundle for this week

本周作业合集

exercises.lol

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1. Polynomial and Rational Functions

Starter Quiz · 课前小测

AP Precalculus Name: ____________________ Score: ________

1. A function is **increasing** on an interval when…
   - ☐ as the input increases, the output increases
   - ☐ as the input increases, the output decreases
   - ☐ the output is a large number
   - ☐ the graph is a straight line

2. The **average rate of change** of f over the interval $[a, b]$ equals…
   - ☐ $f(b) - f(a)$
   - ☐ $\dfrac{f(b) - f(a)}{b - a}$
   - ☐ $\dfrac{f(b)}{f(a)}$
   - ☐ $b - a$

3. The average rate of change of a **linear function** over any interval is…
   - ☐ always zero
   - ☐ always the same (constant)
   - ☐ always increasing
   - ☐ impossible to find

4. The **degree** of a polynomial is…
   - ☐ the number of terms it has
   - ☐ the highest power of x
   - ☐ the constant term
   - ☐ the number of turning points

5. A zero where the graph just **touches** the x-axis (instead of crossing) has an even ______.

   ____

6. An **odd-degree** polynomial with a **positive** leading coefficient…
   - ☐ falls on the left, rises on the right
   - ☐ rises on both ends
   - ☐ falls on both ends
   - ☐ rises on the left, falls on the right

7. If a graph is perfectly flat (horizontal) over an interval, the function is **constant** there.

   ☐ True ☐ False

8. A runner's distance is $f(t)$ metres. If $f(2) = 10$ and $f(5) = 40$, what is the average speed over $[2, 5]$?

   ________________ m/s

9. For a **quadratic function**, the average rate of change is the same over every interval.

   ☐ True ☐ False

10. A point where the graph switches from decreasing to increasing is a local ______.

    ____

1. **as the input increases, the output increases**

   Increasing is about **direction**, not size: move right along the input and the output climbs. It can be increasing while its values are still negative.

2. $\dfrac{f(b) - f(a)}{b - a}$

   It is the **change in output** divided by the **change in input** — a ratio, so it carries units of output per unit of input.

3. **always the same (constant)**

   A straight line has one fixed slope, so the average rate of change is the same over *every* interval — that is what makes it linear.

4. **the highest power of x**

   The degree is the highest exponent on the variable — e.g. $2x^4 - x + 7$ has degree 4. It controls the graph's overall shape and end behavior.

5. **multiplicity**

   Even **multiplicity** (like $(x-3)^2$) makes the curve bounce off the axis; odd multiplicity makes it cross.

6. **falls on the left, rises on the right**

   Odd degree makes the ends point *opposite* ways; a positive lead sends the right end up (and so the left end down).

7. **True**

   A flat graph means the output does not change as the input changes — that is exactly what constant means.

8. **10 m/s**

   Average speed $= \dfrac{40-10}{5-2} = \dfrac{30}{3} = 10$ m/s — the secant slope over that interval.

9. **False**

   A parabola bends, so its slope keeps changing — different intervals give different average rates of change.

10. **minimum**

    A **local minimum** is a valley — the curve stops falling and starts rising.

1.1 Change in Tandem

Name: ____________ Class: ________ Date: __________ **Total: 9 marks**

KEY WORDS function 函数 • domain 定义域 • range 值域 • dependent variable 自变量与因变量 • independent variable 自变量

Objective

Build the skills to answer exam questions on **functions and change in tandem**.

You must be able to:

- define a **function** 函数 as mapping each input to exactly one output
- identify the **domain** 定义域 and **range** 值域
- distinguish the **independent** and **dependent variable** 自变量与因变量

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Functions

A **function** maps each input value to **exactly one** output value. The set of inputs is the **domain**; the set of outputs is the **range**.

■ Variables and the rule

The **independent variable** (x) is the input; the **dependent variable** (y) is the output. Input and output vary **in tandem** according to the **function rule**, which can be given graphically, numerically, analytically, or verbally.

2 Practice

2.1 Define a function. [1]

2.2 State the difference between the domain and the range. [2]

2.3 For $f(x) = 2x + 1$, find $f(3)$. [1]

3 Exam-style questions

3.1 A function maps each input to [1]

- **A** many outputs
- **B** exactly one output
- **C** no output
- **D** two outputs

3.2 The set of input values of a function is its [1]

- **A** range
- **B** domain
- **C** rule
- **D** graph

3.3 $f(x) = x^2$ for x in $\{1, 2, 3\}$.

(a) Find $f(2)$. [1]

(b) State the range. [1]

(c) Name the independent variable. [1]

Solutions

2.1 a relation that maps each input value to exactly one output value.

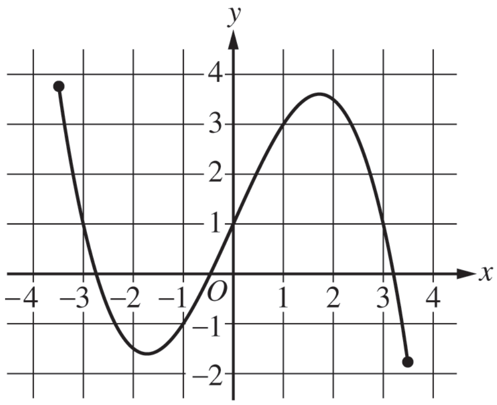
2.2 the domain is the set of input values; the range is the set of output values.

2.3 $f(3) = 2(3) + 1 = 7$.

3.1 **B**.

3.2 **B**.

3.3 (a) $f(2) = 4$. (b) $\{1, 4, 9\}$. (c) x.



Graph of f

1. The figure shows the graph of the function f on its domain of $-3.5 \le x \le 3.5$. The points $(-3, 1)$, $(0, 1)$, and $(3, 1)$ are on the graph of f. The function g is given by $g(x) = 2.916 \cdot (0.7)^x$.

(A) (i) The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(3)$ as a decimal approximation, or indicate that it is not defined.

(ii) Find all values of x for which $f(x) = 1$, or indicate that there are no such values.

(B) (i) Find all values of x, as decimal approximations, for which $g(x) = 2$, or indicate that there are no such values.

(ii) Determine the end behavior of g as x increases without bound. Express your answer using the mathematical notation of a limit.

(C) (i) Determine if f has an inverse function.

(ii) Give a reason for your answer based on the definition of a function and the graph of $y = f(x)$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

AP PRECALCULUS • EXERCISE SHEET

1.2 Rates of Change

Name: ______________ Class: __________ Date: ____________ **Total: 9 marks**

KEY WORDS average rate of change 平均变化率 • degree 次数 • domain 定义域 • range 值域

Objective

Build the skills to answer exam questions on **rates of change**.

You must be able to:

- calculate the **average rate of change 平均变化率** over an interval
- describe the **rate of change at a point 某点变化率**

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Average rate of change

Over an interval $[a, b]$:

$$\text{AROC} = \frac{f(b) - f(a)}{b - a},$$

the ratio of the change in output to the change in input.

■ Rate of change at a point

The rate of change **at a point** describes how fast the output would change there; it can be **approximated** by average rates over small intervals containing the point.

For $f(1) = 3$, $f(4) = 12$: $\text{AROC} = \dfrac{12-3}{4-1} = 3$.

2 Practice

2.1 Write the formula for the average rate of change over $[a, b]$. [1]

2.2 If $f(2) = 5$ and $f(6) = 17$, find the average rate of change over $[2, 6]$. [2]

2.3 State what the rate of change at a point describes. [1]

3 Exam-style questions

3.1 The average rate of change of f over $[a, b]$ is [1]

- **A** $f(b) - f(a)$
- **B** $\dfrac{f(b) - f(a)}{b - a}$
- **C** $b - a$
- **D** $f(b) + f(a)$

3.2 The rate of change at a point can be approximated using [1]

- **A** the range
- **B** average rates over small intervals
- **C** the domain
- **D** the degree

3.3 $f(0) = 2$ and $f(5) = 22$, where f is distance (m) and x is time (s).

(a) Find the average rate of change over $[0, 5]$. [2]

(b) State its units interpretation. [1]

Solutions

2.1 $\dfrac{f(b) - f(a)}{b - a}$.

2.2 $\dfrac{17 - 5}{6 - 2} = \dfrac{12}{4} = 3$.

2.3 how fast the output would change at that input value.

3.1 **B**.

3.2 **B**.

3.3 (a) $\dfrac{22 - 2}{5 - 0} = 4$. (b) 4 metres per second (average speed).

2. A person purchased a car at the end of the year 2019 ($t = 0$). The car's value decreases over time. At the end of the year 2020 ($t = 1$), the car was valued at 27.2 thousand dollars. At the end of the year 2025 ($t = 6$), the car was valued at 14.8 thousand dollars.

The value of the car can be modeled by the function V given by $V(t) = ab^t$, where $V(t)$ is the value of the car, in thousands of dollars, at time t, and t is the number of years since the end of 2019.

A.

i. Use the given data to write two equations that can be used to find the values for constants a and b in the expression for $V(t)$.

ii. Find the values for a and b as decimal approximations.

B.

i. Use the given data to find the average rate of change of the value of the car, in thousands of dollars per year, from $t = 1$ to $t = 6$ years. Express your answer as a decimal approximation. Show the computations that lead to your answer.

ii. Use the average rate of change found in part B (i) to estimate the value of the car, in thousands of dollars, at $t = 3$ years. Show the work that leads to your answer.

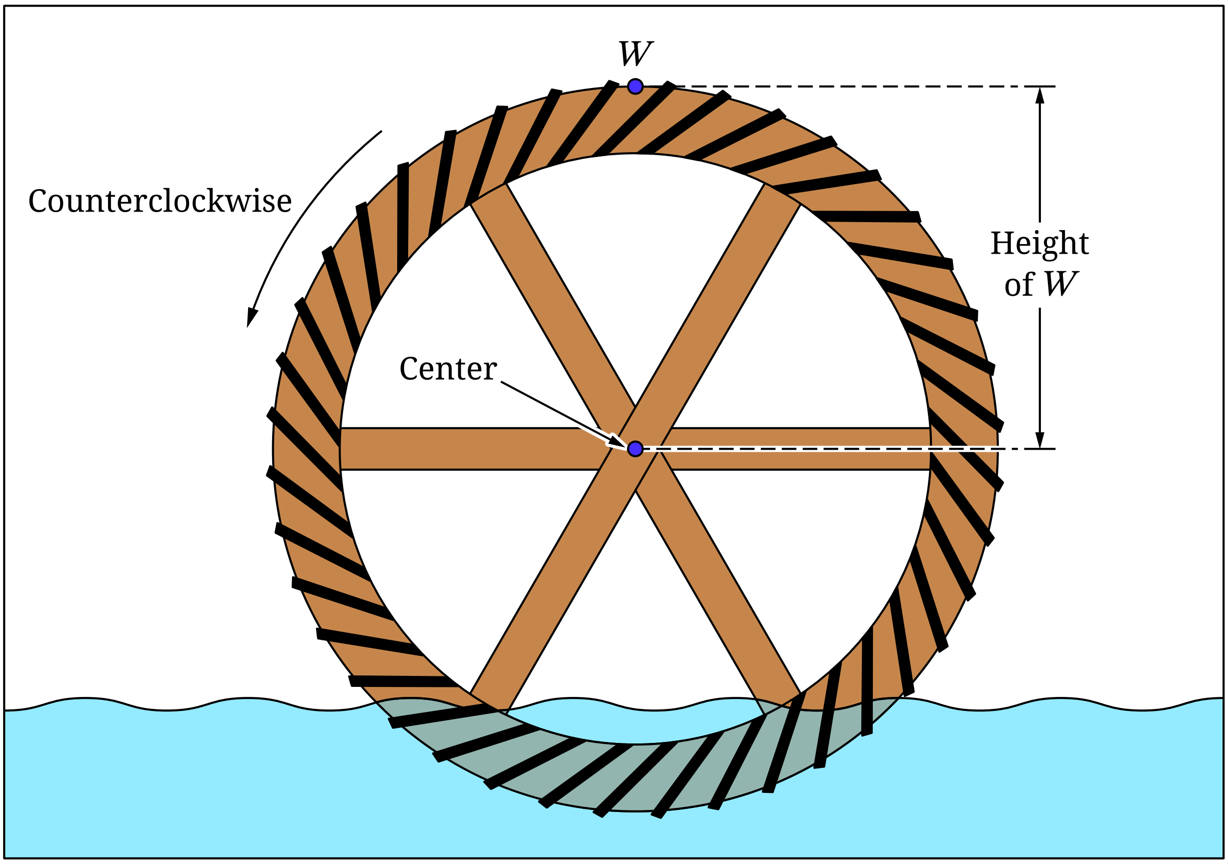
iii. The average rate of change found in part B (i) can be used to determine the secant line for the graph of V on the interval $1 \le t \le 6$. Let $A(t)$ represent the estimate of the value of the car, in thousands of dollars, at time t years, using the secant line. For $A(3)$, found in part B (ii), it can be shown that $A(3) > V(3)$.

In general, $A(t) > V(t)$ for all t, where $1 < t < 6$. Use the secant line and the graph of V to explain why this is true.

C. When the value of the car reaches 2 thousand dollars, the car's owner plans to donate it to an auto mechanic school. As a result, the car will immediately lose all of its value. Explain how this information can be used to determine a domain limitation for the model V.

END OF PART A

3. The figure shows a circular waterwheel that rotates in a counterclockwise direction at a constant rate and completes 1 revolution in 10 seconds. At time $t = 0$, point W on the edge of the waterwheel is at a height of 6 feet directly above the center of the waterwheel. The height of point W from the horizontal line through the center of the waterwheel periodically decreases and increases as the waterwheel moves.

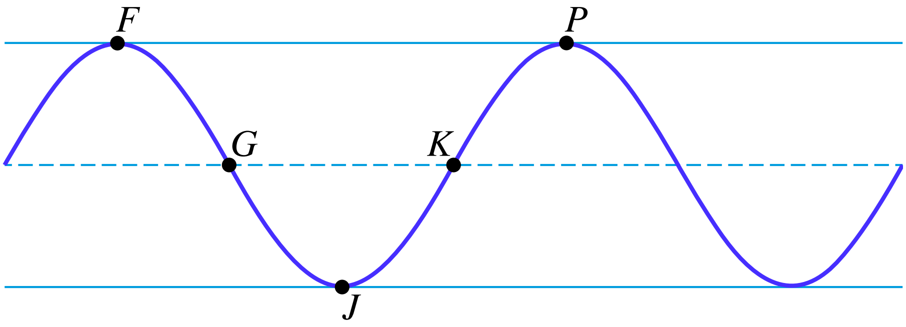


Note: Figure not drawn to scale.

The sinusoidal function h models the height of point W from the horizontal line through the center of the waterwheel, in feet, as a function of time t, in seconds. A positive value of $h(t)$ indicates W is above the line through the center; a negative value of $h(t)$ indicates W is below the line through the center.

A. The graph of h and its dashed midline for two full cycles is shown. Five points, F, G, J, K, and P, are labeled on the graph. No scale is indicated, and no axes are presented.

Determine possible coordinates $(t, h(t))$ for the five points: F, G, J, K, and P.



B. The function h can be written in the form $h(t) = a\sin(b(t+c)) + d$. Find values of constants a, b, c, and d.

C. Refer to the graph of h in part A. The t-coordinate of J is t_1, and the t-coordinate of K is t_2.

i. On the interval (t_1, t_2), which of the following is true about h?

a. h is positive and increasing.

b. h is positive and decreasing.

c. h is negative and increasing.

d. h is negative and decreasing.

ii. On the interval (t_1, t_2), describe the concavity of the graph of h and determine whether the rate of change of h is increasing or decreasing.

2. A musician released a new song on a streaming service. A streaming service is an online entertainment source that allows users to play music on their computers and mobile devices.

Several months later, the musician began using an app (at time $t = 0$) that counts the total number of plays for the song since its release. A "play" is a single stream of the song on the streaming service. The table gives the total number of plays, in thousands, for selected times t months after the musician began using the app. At $t = 0$, the total number of plays was 25 thousand. At $t = 2$, the total number of plays was 30 thousand. At $t = 4$, the total number of plays was 34 thousand.

Months after the musician began using the app	0	2	4
Total number of plays for the song since its release (thousands)	25	30	34

The total number of plays, in thousands, for the song since its release can be modeled by the function D given by $D(t) = at^2 + bt + c$, where $D(t)$ is the total number of plays, in thousands, for the song since its release, and t is the number of months after the musician began using the app.

A.

i. Use the given data to write three equations that can be used to find the values for constants a, b, and c in the expression for $D(t)$.

ii. Find the values for a, b, and c as decimal approximations.

B.

i. Use the given data to find the average rate of change of the total number of plays for the song, in thousands per month, from $t = 0$ to $t = 4$ months. Express your answer as a decimal approximation. Show the computations that lead to your answer.

ii. Use the average rate of change found in part B (i) to estimate the total number of plays for the song, in thousands, for $t = 1.5$ months. Show the work that leads to your answer.

iii. Let A_t represent the estimate of the total number of plays for the song, in thousands, using the average rate of change found in part B (i). For $A_{1.5}$ found in part B (ii), it can be shown that $A_{1.5} < D(1.5)$.
Explain why, in general, $A_t < D(t)$ for all t, where $0 < t < 4$. Your explanation should include a reference to the graph of D and its relationship to A_t.

C. The quadratic function model D has exactly one absolute minimum or one absolute maximum. That minimum or maximum can be used to determine a domain restriction for D.

Based on the context of the problem, explain how that minimum or maximum can be used to determine a boundary for the domain of D.

END OF PART A

2. On the initial day of sales ($t = 0$) for a new video game, there were 40 thousand units of the game sold that day. Ninety-one days later ($t = 91$), there were 76 thousand units of the game sold that day.

   The number of units of the video game sold on a given day can be modeled by the function G given by $G(t) = a + b\ln(t + 1)$, where $G(t)$ is the number of units sold, in thousands, on day t since the initial day of sales.

   (A) (i) Use the given data to write two equations that can be used to find the values for constants a and b in the expression for $G(t)$.

   (ii) Find the values for a and b as decimal approximations.

   (B) (i) Use the given data to find the average rate of change of the number of units of the video game sold, in thousands per day, from $t = 0$ to $t = 91$ days. Express your answer as a decimal approximation. Show the computations that lead to your answer.

   (ii) Use the average rate of change found in (i) to estimate the number of units of the video game sold, in thousands, on day $t = 50$. Show the work that leads to your answer.

   (iii) Let A_t represent the estimate of the number of units of the video game sold, in thousands, using the average rate of change found in (i). For A_{50}, found in (ii), it can be shown that $A_{50} < G(50)$. Explain why, in general, $A_t < G(t)$ for all t, where $0 < t < 91$.

   (C) The makers of the video game reported that daily sales of the video game decreased each day after $t = 91$. Explain why the error in the model G increases after $t = 91$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

END OF PART A

PRECALCULUS

SECTION II, Part B

Time—30 minutes

2 Questions

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.

1.3 Rates of Change in Linear and Quadratic Functions

Name: ______________ Class: __________ Date: ____________ **Total: 9 marks**

KEY WORDS linear function 线性函数 • quadratic 二次 • quadratic function's 二次函数 • zero 零点 • function 函数

Objective

Build the skills to answer exam questions on **rates of change in linear and quadratic functions**.

You must be able to:

- explain that a **linear function 线性函数** has a **constant** average rate of change
- explain that a **quadratic function's 二次函数** average rates of change follow a **linear** pattern

1 Worked examples

Study these first. Each one shows the method for a question type used later.

Linear functions

Over **any** interval, a linear function has the **same** (constant) average rate of change — its slope.

Quadratic functions

For a quadratic, the average rates of change over **consecutive equal-length** intervals are **not** constant, but they change in a **linear** pattern (a constant second difference).

Example

A linear function with $f(0) = 1$, $f(2) = 7$: AROC $= \dfrac{7-1}{2-0} = 3$ over every interval.

2 Practice

2.1 State the average-rate-of-change property of a linear function. [1]

2.2 State how the average rates of change of a quadratic behave over equal intervals. [1]

2.3 A linear function has $f(0) = 1$ and $f(2) = 7$. Find its (constant) average rate of change. [2]

3 Exam-style questions

3.1 A linear function has an average rate of change that is [1]

- **A** always increasing
- **B** constant
- **C** always zero
- **D** quadratic

3.2 For a quadratic, the average rates of change over equal intervals form a [1]

- **A** constant sequence
- **B** linear pattern
- **C** quadratic pattern
- **D** random sequence

3.3 A linear function passes through $(1, 4)$ and $(3, 10)$.

(a) Find the average rate of change. [2]

(b) State whether it changes over other intervals. [1]

Solutions

2.1 it is constant over any interval.

2.2 they change in a linear pattern.

2.3 $\dfrac{7-1}{2-0} = 3.$

3.1 **B**.

3.2 **B**.

3.3 (a) $\dfrac{10-4}{3-1} = 3.$ (b) no — it is constant.

2. A musician released a new song on a streaming service. A streaming service is an online entertainment source that allows users to play music on their computers and mobile devices.

Several months later, the musician began using an app (at time $t = 0$) that counts the total number of plays for the song since its release. A "play" is a single stream of the song on the streaming service. The table gives the total number of plays, in thousands, for selected times t months after the musician began using the app. At $t = 0$, the total number of plays was 25 thousand. At $t = 2$, the total number of plays was 30 thousand. At $t = 4$, the total number of plays was 34 thousand.

Months after the musician began using the app	0	2	4
Total number of plays for the song since its release (thousands)	25	30	34

The total number of plays, in thousands, for the song since its release can be modeled by the function D given by $D(t) = at^2 + bt + c$, where $D(t)$ is the total number of plays, in thousands, for the song since its release, and t is the number of months after the musician began using the app.

A.

i. Use the given data to write three equations that can be used to find the values for constants a, b, and c in the expression for $D(t)$.

ii. Find the values for a, b, and c as decimal approximations.

B.

i. Use the given data to find the average rate of change of the total number of plays for the song, in thousands per month, from $t = 0$ to $t = 4$ months. Express your answer as a decimal approximation. Show the computations that lead to your answer.

ii. Use the average rate of change found in part B (i) to estimate the total number of plays for the song, in thousands, for $t = 1.5$ months. Show the work that leads to your answer.

iii. Let A_t represent the estimate of the total number of plays for the song, in thousands, using the average rate of change found in part B (i). For $A_{1.5}$ found in part B (ii), it can be shown that $A_{1.5} < D(1.5)$.
Explain why, in general, $A_t < D(t)$ for all t, where $0 < t < 4$. Your explanation should include a reference to the graph of D and its relationship to A_t.

C. The quadratic function model D has exactly one absolute minimum or one absolute maximum. That minimum or maximum can be used to determine a domain restriction for D.

Based on the context of the problem, explain how that minimum or maximum can be used to determine a boundary for the domain of D.

END OF PART A

1.4 Polynomial Functions and Rates of Change

Name: ____________ Class: ________ Date: __________ **Total: 9 marks**

KEY WORDS local 局部 • degree 次数 • global 全局 • leading coefficient 首项系数 • polynomial 多项式

Objective

Build the skills to answer exam questions on **polynomial functions and rates of change**.

You must be able to:

- identify the **degree 次数**, **leading term 首项**, and **leading coefficient 首项系数** of a polynomial
- identify **local 局部** and **global 全局** maxima and minima

1 Worked examples

Study these first. Each one shows the method for a question type used later.

Polynomial functions

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0.$$

Its **degree** is n, its **leading term** is $a_n x^n$, and its **leading coefficient** is a_n.

Local and global extrema

Where a polynomial switches between increasing and decreasing, it has a **local** (relative) maximum or minimum. The greatest local maximum is the **global** (absolute) maximum; the least local minimum is the global minimum.

2 Practice

2.1 State the degree and leading coefficient of $p(x) = 3x^4 - 2x + 5$. [2]

2.2 Define a local maximum. [1]

2.3 State the difference between a local and a global maximum. [1]

3 Exam-style questions

3.1 The degree of $p(x) = 5x^3 + 2x^2 - 1$ is [1]

- **A** 5
- **B** 3
- **C** 2
- **D** 1

3.2 A polynomial switches from increasing to decreasing at a [1]

- **A** zero
- **B** local maximum
- **C** asymptote
- **D** hole

3.3 $p(x) = 2x^5 - 7x + 4$.

(a) State the degree. [1]

(b) State the leading term. [1]

(c) State the leading coefficient. [1]

Solutions

2.1 degree 4; leading coefficient 3.

2.2 an output value greater than those at nearby inputs (where the function switches from increasing to decreasing).

2.3 a local maximum is greatest among nearby points; the global maximum is the greatest of all local maxima.

3.1 **B**.

3.2 **B**.

3.3 (a) 5. (b) $2x^5$. (c) 2.

1.5 Polynomial Functions and Complex Zeros

Name: ____________ Class: ________ Date: __________ **Total: 9 marks**

KEY WORDS zero 零点 • multiplicity 重数 • complex 复数 • function 函数 • degree 次数

Objective

Build the skills to answer exam questions on **polynomial functions and complex zeros**.

You must be able to:

- relate a **zero** 零点 a to the linear factor $(x-a)$
- describe the **multiplicity** 重数 of a zero
- use that a degree-n polynomial has exactly n complex zeros (with multiplicity)

1 Worked examples

Study these first. Each one shows the method for a question type used later.

Zeros and factors

If $p(a)=0$ then a is a **zero** of p, and (for real a) $(x-a)$ is a **factor**.

Multiplicity

If a factor $(x-a)$ is repeated n times, the zero has **multiplicity** n. A polynomial of degree n has exactly n complex zeros, counting multiplicities.

Example

$p(x)=(x-2)^2(x+1)$ has zeros $x=2$ (multiplicity 2) and $x=-1$ (multiplicity 1); degree 3.

2 Practice

2.1 State the relationship between a zero a and the factor $(x-a)$. [1]

2.2 State how many complex zeros a degree-4 polynomial has (counting multiplicity). [1]

2.3 For $p(x) = (x-2)^2(x+1)$, state the zeros and their multiplicities. [2]

3 Exam-style questions

3.1 If $p(a) = 0$, then $(x - a)$ is a [1]

- **A** vertical asymptote
- **B** factor of p
- **C** hole
- **D** leading term

3.2 A degree-5 polynomial has _________ complex zeros counting multiplicity. [1]

- **A** 1
- **B** 3
- **C** 5
- **D** 10

3.3 $p(x) = (x-3)^3(x+2)$.

(a) State the zeros. [1]

(b) State the multiplicity of $x = 3$. [1]

(c) State the degree. [1]

Solutions

2.1 $(x - a)$ is a factor of p if and only if a is a zero of p.

2.2 4.

2.3 $x = 2$ (multiplicity 2); $x = -1$ (multiplicity 1).

3.1 **B**.

3.2 **C**.

3.3 (a) $x = 3$ and $x = -2$. (b) 3. (c) 4.

1. The function f is decreasing and is defined for all real numbers. The table gives values for $f(x)$ at selected values of x.

x	-2	-1	0	1	2
$f(x)$	14	7	3.5	1.75	0.875

The function g is given by $g(x) = -0.167x^3 + x^2 - 1.834$.

A.

i. The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(1)$ as a decimal approximation, or indicate that it is not defined. Show the work that leads to your answer.

ii. Find the value of $f^{-1}(3.5)$, or indicate that it is not defined.

B.

i. Find all values of x, as decimal approximations, for which $g(x) = 0$, or indicate that there are no such values.

ii. Determine the end behavior of g as x increases without bound. Express your answer using the mathematical notation of a limit.

C.

i. Based on the table, which of the following function types best models function f: linear, quadratic, exponential, or logarithmic?

ii. Give a reason for your answer in part C (i) based on the relationship between the change in the output values of f and the change in the input values of f. Refer to the values in the table in your reasoning.

1.6 Polynomial Functions and End Behavior

Name: ____________ Class: ________ Date: __________ **Total: 9 marks**

KEY WORDS end behavior 端点行为 • limit notation 极限记号 • function 函数 • polynomial 多项式 • leading term 首项

Objective

Build the skills to answer exam questions on **polynomial functions and end behavior**.

You must be able to:

- describe the **end behavior** 端点行为 of a polynomial from its leading term
- use **limit notation** 极限记号 such as $\lim_{x\to\infty} p(x) = \infty$

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ End behavior

The **end behavior** of a polynomial is set by its **leading term** $a_n x^n$:

- **even** n, $a_n > 0$: both ends up.
- **odd** n, $a_n > 0$: left end down, right end up.

■ Limit notation

"Output increases without bound as x increases" is written $\lim_{x\to\infty} p(x) = \infty$. For $p(x) = x^3$: $\lim_{x\to\infty} p(x) = \infty$ and $\lim_{x\to-\infty} p(x) = -\infty$.

2 Practice

2.1 State what determines a polynomial's end behavior. [1]

2.2 For $p(x) = x^3$, state the end behavior as $x \to \infty$ and as $x \to -\infty$. [2]

2.3 Write the limit notation for ”output increases without bound as x increases”. [1]

---

3 Exam-style questions

3.1 The end behavior of a polynomial is determined by its [1]

- **A** constant term
- **B** leading term
- **C** middle term
- **D** zero

3.2 For $p(x) = -2x^4$, as $x \to \infty$ the output [1]

- **A** $\to \infty$
- **B** $\to -\infty$
- **C** $\to 0$
- **D** oscillates

3.3 $p(x) = x^3$.

(a) State $\lim_{x\to\infty} p(x)$. [1]

(b) State $\lim_{x\to-\infty} p(x)$. [1]

(c) State whether both ends go the same way. [1]

Solutions

2.1 its leading term (degree and leading coefficient).

2.2 as $x \to \infty$, $p(x) \to \infty$; as $x \to -\infty$, $p(x) \to -\infty$.

2.3 $\lim_{x\to\infty} p(x) = \infty$.

3.1 **B**.

3.2 **B** — an even degree with a negative leading coefficient sends both ends to $-\infty$.

3.3 (a) ∞. (b) $-\infty$. (c) no (opposite directions).

1. The function f is decreasing and is defined for all real numbers. The table gives values for $f(x)$ at selected values of x.

x	-2	-1	0	1	2
$f(x)$	14	7	3.5	1.75	0.875

The function g is given by $g(x) = -0.167x^3 + x^2 - 1.834$.

A.

i. The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(1)$ as a decimal approximation, or indicate that it is not defined. Show the work that leads to your answer.

ii. Find the value of $f^{-1}(3.5)$, or indicate that it is not defined.

B.

i. Find all values of x, as decimal approximations, for which $g(x) = 0$, or indicate that there are no such values.

ii. Determine the end behavior of g as x increases without bound. Express your answer using the mathematical notation of a limit.

C.

i. Based on the table, which of the following function types best models function f: linear, quadratic, exponential, or logarithmic?

ii. Give a reason for your answer in part C (i) based on the relationship between the change in the output values of f and the change in the input values of f. Refer to the values in the table in your reasoning.

1.7 Rational Functions and End Behavior

Name: ____________ Class: ________ Date: __________ **Total: 9 marks**

KEY WORDS horizontal asymptote 水平渐近线 • rational function 有理函数 • quotient 商 • degree 次数 • end behavior 末端行为

Objective

Build the skills to answer exam questions on **rational functions and end behavior**.

You must be able to:

- describe a **rational function 有理函数** as a quotient of polynomials
- find its end behavior from the ratio of the **leading terms**
- identify a **horizontal asymptote 水平渐近线** where one exists

1 Worked examples

Study these first. Each one shows the method for a question type used later.

Rational functions

A **rational function** $r(x) = \dfrac{p(x)}{q(x)}$ is a quotient of two polynomials.

End behavior

For large $|x|$ each polynomial is dominated by its leading term, so the end behavior is the ratio of the leading terms:

- degree(top) < degree(bottom) → horizontal asymptote $y = 0$;
- degrees **equal** → $y = \dfrac{\text{leading coeff of top}}{\text{leading coeff of bottom}}$;
- degree(top) > degree(bottom) → **no** horizontal asymptote.

2 Practice

2.1 State what determines the end behavior of a rational function. [1]

2.2 For $r(x) = \dfrac{2x}{x^2+1}$, state the horizontal asymptote. [1]

2.3 For $r(x) = \dfrac{3x^2}{x^2+1}$, state the horizontal asymptote. [2]

3 Exam-style questions

3.1 The end behavior of a rational function is found from the ratio of the [1]

- **A** constant terms
- **B** leading terms
- **C** zeros
- **D** holes

3.2 If the numerator degree is less than the denominator degree, the horizontal asymptote is [1]

- **A** $y = 1$
- **B** $y = 0$
- **C** none
- **D** $y = x$

3.3 $r(x) = \dfrac{4x^2+1}{2x^2-3}$.

(a) State the horizontal asymptote. [2]

(b) State whether the degrees are equal. [1]

Solutions

2.1 the ratio of the leading terms of the numerator and denominator.

2.2 $y = 0$ (numerator degree is smaller).

2.3 $y = 3$ (equal degrees, ratio of leading coefficients 3/1).

3.1 **B**.

3.2 **B**.

3.3 (a) $y = \frac{4}{2} = 2$. (b) yes (both degree 2).

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1.8 Rational Functions and Zeros

Name: ______________ Class: __________ Date: ____________ **Total: 8 marks**

KEY WORDS zeros 零点 • domain 定义域 • function 函数 • rational function 有理函数 • hole 空洞

Objective

Build the skills to answer exam questions on **rational functions and zeros**.

You must be able to:

- find the real **zeros** 零点 of a rational function from its numerator
- recognise which numerator zeros are excluded because they are outside the domain

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Zeros of a rational function

The real zeros of $r(x) = \dfrac{p(x)}{q(x)}$ are the real zeros of the **numerator** $p(x)$ — provided those inputs are **in the domain** (i.e. the denominator is not also zero there).

■ Example

$r(x) = \dfrac{x-4}{x+1}$ has a zero at $x = 4$ (and $x = -1$ is excluded from the domain).

2 Practice

2.1 State where the real zeros of a rational function come from. [1]

2.2 For $r(x) = \dfrac{x-4}{x+1}$, state the zero. [1]

2.3 State why a numerator zero might **not** be a zero of r. [1]

3 Exam-style questions

3.1 The real zeros of a rational function come from the [1]

- **A** denominator
- **B** numerator
- **C** leading term
- **D** degree

3.2 The function $r(x) = \dfrac{x-2}{x-2}$ [1]

- **A** has a zero at 2
- **B** has a hole at 2
- **C** has an asymptote at 2
- **D** is undefined everywhere

3.3 $r(x) = \dfrac{x^2-9}{x-1}$.

(a) State the zeros of the numerator. [1]

(b) State the zeros of r. [1]

(c) State the value excluded from the domain. [1]

Solutions

2.1 from the real zeros of the numerator (that lie in the domain).

2.2 $x = 4$.

2.3 if it is also a zero of the denominator, it is outside the domain (a hole, not a zero).

3.1 **B**.

3.2 **B**.

3.3 (a) $x = 3$ and $x = -3$. (b) $x = 3$ and $x = -3$. (c) $x = 1$.

1.9 Rational Functions and Vertical Asymptotes

Name: ______________ Class: __________ Date: ____________ **Total: 8 marks**

KEY WORDS vertical asymptote 垂直渐近线 • function 函数 • zero 零点 • rational function 有理函数 • leading term 首项

Objective

Build the skills to answer exam questions on **rational functions and vertical asymptotes**.

You must be able to:

- identify a **vertical asymptote 垂直渐近线** at $x = a$
- describe the behaviour of $r(x)$ near a vertical asymptote

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Vertical asymptotes

A rational function has a **vertical asymptote** at $x = a$ if a is a zero of the **denominator** but **not** of the numerator (or has higher multiplicity in the denominator).

■ Behaviour near it

Near $x = a$, the denominator is close to 0, so the values **increase or decrease without bound**: $\lim_{x\to a^+} r(x) = \pm\infty$.

For $r(x) = \dfrac{1}{x-3}$, the vertical asymptote is $x = 3$.

2 Practice

2.1 State the condition for a vertical asymptote at $x = a$. [1]

2.2 For $r(x) = \dfrac{1}{x-3}$, state the vertical asymptote. [1]

2.3 State what happens to $r(x)$ near a vertical asymptote. [1]

---

3 Exam-style questions

3.1 A rational function has a vertical asymptote where the ________ is zero (but not the numerator). [1]

- **A** numerator
- **B** denominator
- **C** leading term
- **D** degree

3.2 Near a vertical asymptote, the function values [1]

- **A** approach 0
- **B** increase or decrease without bound
- **C** stay constant
- **D** equal the mean

3.3 $r(x) = \dfrac{x+2}{x-5}$.

(a) State the vertical asymptote. [1]

(b) State the zero. [1]

(c) State the domain restriction. [1]

Solutions

2.1 a is a zero of the denominator but not of the numerator.

2.2 $x = 3$.

2.3 the values increase or decrease without bound.

3.1 **B**.

3.2 **B**.

3.3 (a) $x = 5$. (b) $x = -2$. (c) $x \neq 5$.

1.10 Rational Functions and Holes

Name: ______________ Class: __________ Date: ____________ **Total: 9 marks**

KEY WORDS hole 空心点 • function 函数 • rational function 有理函数 • vertical asymptote 垂直渐近线 • leading term 首项

Objective

Build the skills to answer exam questions on **rational functions and holes**.

You must be able to:

- identify a **hole 空心点** where a factor cancels from numerator and denominator
- find the location (c, L) of a hole

1 Worked examples

Study these first. Each one shows the method for a question type used later.

Holes

A rational function has a **hole** at $x = c$ when a factor $(x - c)$ **cancels** — the numerator's multiplicity of c is at least the denominator's.

Location

The hole is at (c, L), where $L = \lim_{x\to c} r(x)$ — evaluate the **simplified** function at c.

Example

$r(x) = \dfrac{(x-2)(x+1)}{x-2}$ simplifies to $x + 1$ (for $x \neq 2$), so there is a hole at $(2, 3)$.

2 Practice

2.1 State when a rational function has a hole. [1]

2.2 For $r(x) = \dfrac{(x-2)(x+1)}{x-2}$, state the location of the hole. [2]

2.3 State the difference between a hole and a vertical asymptote. [1]

3 Exam-style questions

3.1 A rational function has a hole where a factor [1]

- **A** appears only in the denominator
- **B** cancels from numerator and denominator
- **C** is the leading term
- **D** is constant

3.2 The x-value of a hole is [1]

- **A** always 0
- **B** where a common factor is zero
- **C** a vertical asymptote
- **D** undefined for all x

3.3 $r(x) = \dfrac{(x-3)(x+4)}{x-3}$.

(a) Simplify $r(x)$ (for $x \neq 3$). [1]

(b) State the x-coordinate of the hole. [1]

(c) Find the y-coordinate of the hole. [1]

Solutions

2.1 when a factor cancels from both the numerator and the denominator.

2.2 simplifies to $x + 1$; hole at $(2, 3)$.

2.3 at a hole the function is merely undefined at one point; at an asymptote the values blow up to $\pm\infty$.

3.1 **B**.

3.2 **B**.

3.3 (a) $x + 4$. (b) $x = 3$. (c) $3 + 4 = 7$.

1.11 Equivalent Representations of Polynomial and Rational Expressions

Name: ____________ Class: __________ Date: ____________ **Total: 9 marks**

KEY WORDS factored form 因式分解形式 • standard form 标准形式 • end behavior 末端行为 • hole 空洞 • zero 零点

Objective

Build the skills to answer exam questions on **equivalent representations of polynomial and rational expressions**.

You must be able to:

- use the **factored form** 因式分解形式 to read off zeros, intercepts, asymptotes, and holes
- use the **standard form** 标准形式 to read off end behavior

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Two useful forms

- **Factored form** shows the **real zeros** directly, revealing x-intercepts, vertical asymptotes, holes, and domain.
- **Standard form** (expanded) shows the **leading term**, revealing **end behavior**.

■ Example

$p(x) = (x - 1)(x + 3)$ has x-intercepts at 1 and -3; expanded, $p(x) = x^2 + 2x - 3$ shows an upward parabola (even degree, positive leading coefficient).

2 Practice

2.1 State what the factored form of a function reveals. [1]

2.2 State what the standard (expanded) form reveals. [1]

2.3 For $p(x) = (x-1)(x+3)$, state the x-intercepts. [2]

3 Exam-style questions

3.1 The factored form of a polynomial readily shows its [1]

- **A** end behavior
- **B** real zeros / x-intercepts
- **C** leading coefficient only
- **D** nothing useful

3.2 The standard form best reveals a function's [1]

- **A** holes
- **B** end behavior
- **C** x-intercepts
- **D** domain

3.3 $p(x) = (x-2)(x+5)$.

(a) State the x-intercepts. [1]

(b) Expand to standard form. [2]

Solutions

2.1 the real zeros, and hence x-intercepts, asymptotes, holes, and domain.

2.2 the end behavior (via the leading term).

2.3 $x = 1$ and $x = -3$.

3.1 **B**.

3.2 **B**.

3.3 (a) $x = 2$ and $x = -5$. (b) $x^2 + 3x - 10$.

AP PRECALCULUS • EXERCISE SHEET

1.12 Transformations of Functions

Name: ______________ Class: __________ Date: ____________ **Total: 8 marks**

KEY WORDS transformation 变换 • horizontal translation 水平平移 • vertical translation 竖直平移 • translations 平移 • function 函数

Objective

Build the skills to answer exam questions on **transformations of functions**.

You must be able to:

- describe $g(x) = f(x) + k$ as a **vertical translation 竖直平移**
- describe $g(x) = f(x+h)$ as a **horizontal translation 水平平移**

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Translations

- $g(x) = f(x) + k$ shifts the graph **vertically** by k (up if $k > 0$).
- $g(x) = f(x+h)$ shifts the graph **horizontally** by $-h$ (so $f(x-2)$ shifts **right** 2).

■ Example

From $f(x) = x^2$: $g(x) = x^2 + 3$ shifts up 3; $h(x) = (x-1)^2$ shifts right 1, with vertex at $(1, 0)$.

2 Practice

2.1 State the transformation given by $f(x) + 3$. [1]

2.2 State the transformation given by $f(x-2)$. [1]

2.3 Describe the graph of $g(x) = f(x) + 5$ compared with f. [1]

3 Exam-style questions

3.1 $g(x) = f(x) + k$ is a [1]

- **A** horizontal shift
- **B** vertical shift by k
- **C** reflection
- **D** stretch

3.2 $g(x) = f(x - 4)$ shifts the graph of f [1]

- **A** left 4
- **B** right 4
- **C** up 4
- **D** down 4

3.3 Start from $f(x) = x^2$.

(a) Describe $g(x) = x^2 + 3$. [1]

(b) Describe $h(x) = (x - 1)^2$. [1]

(c) State the vertex of h. [1]

Solutions

2.1 a vertical shift up by 3.

2.2 a horizontal shift right by 2.

2.3 it is the graph of f shifted up 5 units.

3.1 **B**.

3.2 **B**.

3.3 (a) shift f up 3. (b) shift f right 1. (c) $(1, 0)$.

1.13 Function Model Selection and Assumption Articulation

Name: ______________ Class: __________ Date: ____________ **Total: 8 marks**

KEY WORDS function model 函数模型 • assumptions 假设 • limitations 局限性 • extrapolation 外推 • function 函数

Objective

Build the skills to answer exam questions on **function model selection and assumption articulation**.

You must be able to:

- select an appropriate **function model 函数模型** (linear, quadratic, polynomial, rational) from how a quantity changes
- articulate the **assumptions 假设** and **limitations 局限性** of a model

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Choosing a model

- **constant** rate of change → linear;
- constant **second difference** → quadratic;
- several turning points → higher-degree polynomial;
- a horizontal asymptote → rational.

■ Assumptions and limitations

State the **domain** over which the model is valid, and note that predictions far outside the data (**extrapolation 外推**) may be invalid.

2 Practice

2.1 State the type of function that has a constant rate of change. [1]

2.2 State the type of function whose graph can have a horizontal asymptote. [1]

2.3 A data set has a constant second difference. State the best model type. [1]

3 Exam-style questions

3.1 A quantity changes at a constant rate. The best model is [1]

- **A** linear
- **B** exponential
- **C** rational
- **D** constant

3.2 A model built from data should state its [1]

- **A** colour
- **B** domain of validity and assumptions
- **C** author
- **D** title

3.3 A population is recorded each year for 5 years, and the yearly increases are equal.

(a) State the best model type. [1]

(b) State one assumption in using it to predict year 20. [1]

(c) State one limitation. [1]

Solutions

2.1 a linear function.

2.2 a rational function.

2.3 quadratic.

3.1 A.

3.2 B.

3.3 (a) linear. (b) that the constant yearly increase continues unchanged. (c) extrapolating far beyond the 5 years of data may be unreliable.

2. A person purchased a car at the end of the year 2019 ($t = 0$). The car's value decreases over time. At the end of the year 2020 ($t = 1$), the car was valued at 27.2 thousand dollars. At the end of the year 2025 ($t = 6$), the car was valued at 14.8 thousand dollars.

The value of the car can be modeled by the function V given by $V(t) = ab^t$, where $V(t)$ is the value of the car, in thousands of dollars, at time t, and t is the number of years since the end of 2019.

A.

i. Use the given data to write two equations that can be used to find the values for constants a and b in the expression for $V(t)$.

ii. Find the values for a and b as decimal approximations.

B.

i. Use the given data to find the average rate of change of the value of the car, in thousands of dollars per year, from $t = 1$ to $t = 6$ years. Express your answer as a decimal approximation. Show the computations that lead to your answer.

ii. Use the average rate of change found in part B (i) to estimate the value of the car, in thousands of dollars, at $t = 3$ years. Show the work that leads to your answer.

iii. The average rate of change found in part B (i) can be used to determine the secant line for the graph of V on the interval $1 \le t \le 6$. Let $A(t)$ represent the estimate of the value of the car, in thousands of dollars, at time t years, using the secant line. For $A(3)$, found in part B (ii), it can be shown that $A(3) > V(3)$.

In general, $A(t) > V(t)$ for all t, where $1 < t < 6$. Use the secant line and the graph of V to explain why this is true.

C. When the value of the car reaches 2 thousand dollars, the car's owner plans to donate it to an auto mechanic school. As a result, the car will immediately lose all of its value. Explain how this information can be used to determine a domain limitation for the model V.

END OF PART A

1. The function f is decreasing and is defined for all real numbers. The table gives values for $f(x)$ at selected values of x.

x	-2	-1	0	1	2
$f(x)$	14	7	3.5	1.75	0.875

The function g is given by $g(x) = -0.167x^3 + x^2 - 1.834$.

A.

i. The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(1)$ as a decimal approximation, or indicate that it is not defined. Show the work that leads to your answer.

ii. Find the value of $f^{-1}(3.5)$, or indicate that it is not defined.

B.

i. Find all values of x, as decimal approximations, for which $g(x) = 0$, or indicate that there are no such values.

ii. Determine the end behavior of g as x increases without bound. Express your answer using the mathematical notation of a limit.

C.

i. Based on the table, which of the following function types best models function f: linear, quadratic, exponential, or logarithmic?

ii. Give a reason for your answer in part C (i) based on the relationship between the change in the output values of f and the change in the input values of f. Refer to the values in the table in your reasoning.

2. A musician released a new song on a streaming service. A streaming service is an online entertainment source that allows users to play music on their computers and mobile devices.

Several months later, the musician began using an app (at time $t = 0$) that counts the total number of plays for the song since its release. A "play" is a single stream of the song on the streaming service. The table gives the total number of plays, in thousands, for selected times t months after the musician began using the app. At $t = 0$, the total number of plays was 25 thousand. At $t = 2$, the total number of plays was 30 thousand. At $t = 4$, the total number of plays was 34 thousand.

Months after the musician began using the app	0	2	4
Total number of plays for the song since its release (thousands)	25	30	34

The total number of plays, in thousands, for the song since its release can be modeled by the function D given by $D(t) = at^2 + bt + c$, where $D(t)$ is the total number of plays, in thousands, for the song since its release, and t is the number of months after the musician began using the app.

A.

i. Use the given data to write three equations that can be used to find the values for constants a, b, and c in the expression for $D(t)$.

ii. Find the values for a, b, and c as decimal approximations.

B.

i. Use the given data to find the average rate of change of the total number of plays for the song, in thousands per month, from $t = 0$ to $t = 4$ months. Express your answer as a decimal approximation. Show the computations that lead to your answer.

ii. Use the average rate of change found in part B (i) to estimate the total number of plays for the song, in thousands, for $t = 1.5$ months. Show the work that leads to your answer.

iii. Let A_t represent the estimate of the total number of plays for the song, in thousands, using the average rate of change found in part B (i). For $A_{1.5}$ found in part B (ii), it can be shown that $A_{1.5} < D(1.5)$.
Explain why, in general, $A_t < D(t)$ for all t, where $0 < t < 4$. Your explanation should include a reference to the graph of D and its relationship to A_t.

C. The quadratic function model D has exactly one absolute minimum or one absolute maximum. That minimum or maximum can be used to determine a domain restriction for D.

Based on the context of the problem, explain how that minimum or maximum can be used to determine a boundary for the domain of D.

END OF PART A

2. On the initial day of sales ($t = 0$) for a new video game, there were 40 thousand units of the game sold that day. Ninety-one days later ($t = 91$), there were 76 thousand units of the game sold that day.

The number of units of the video game sold on a given day can be modeled by the function G given by $G(t) = a + b\ln(t+1)$, where $G(t)$ is the number of units sold, in thousands, on day t since the initial day of sales.

(A) (i) Use the given data to write two equations that can be used to find the values for constants a and b in the expression for $G(t)$.

(ii) Find the values for a and b as decimal approximations.

(B) (i) Use the given data to find the average rate of change of the number of units of the video game sold, in thousands per day, from $t = 0$ to $t = 91$ days. Express your answer as a decimal approximation. Show the computations that lead to your answer.

(ii) Use the average rate of change found in (i) to estimate the number of units of the video game sold, in thousands, on day $t = 50$. Show the work that leads to your answer.

(iii) Let A_t represent the estimate of the number of units of the video game sold, in thousands, using the average rate of change found in (i). For A_{50}, found in (ii), it can be shown that $A_{50} < G(50)$. Explain why, in general, $A_t < G(t)$ for all t, where $0 < t < 91$.

(C) The makers of the video game reported that daily sales of the video game decreased each day after $t = 91$. Explain why the error in the model G increases after $t = 91$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

END OF PART A

PRECALCULUS

SECTION II, Part B

Time—30 minutes

2 Questions

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.

1.14 Function Model Construction and Application

Name: ____________ Class: ________ Date: __________ **Total: 8 marks**

KEY WORDS function model 函数模型 • model 模型 • function 函数 • zero 零点 • leading coefficient 首项系数

Objective

Build the skills to answer exam questions on **function model construction and application**.

You must be able to:

- construct a **function model 函数模型** from zeros, points, and transformations
- apply the model to make a prediction

1 Worked examples

Study these first. Each one shows the method for a question type used later.

Constructing a model

Use the given conditions: **zeros** give factors, one extra **point** fixes the leading coefficient, and stated **transformations** set the shifts.

Example

A parabola with zeros at $x = 1$ and $x = 5$ passing through $(3, -4)$: write $f(x) = a(x-1)(x-5)$. At $x = 3$: $a(2)(-2) = -4a = -4$, so $a = 1$ and $f(x) = (x-1)(x-5)$.

2 Practice

2.1 Write the factored form of a quadratic with zeros at 2 and 6. [1]

2.2 State what extra information fixes the leading coefficient. [1]

2.3 Evaluate $f(x) = (x-1)(x-5)$ at $x = 0$. [1]

3 Exam-style questions

3.1 A quadratic model with zeros at $x = r$ and $x = s$ has the form [1]

- **A** $a(x-r)(x-s)$
- **B** $ax + b$
- **C** a/x
- **D** a^x

3.2 To find the leading coefficient a, you need [1]

- **A** the degree
- **B** one additional point on the graph
- **C** the axis of symmetry
- **D** nothing extra

3.3 A parabola has zeros at $x = 1$ and $x = 5$ and passes through $(3, -4)$.

(a) Write it as $f(x) = a(x-1)(x-5)$ and substitute $(3, -4)$. [1]

(b) Solve for a. [1]

(c) State $f(x)$. [1]

Solutions

2.1 $a(x-2)(x-6)$.

2.2 one additional point that the graph passes through.

2.3 $(0-1)(0-5)=(-1)(-5)=5$.

3.1 **A**.

3.2 **B**.

3.3 (a) $a(3-1)(3-5)=-4$, i.e. $-4a=-4$. (b) $a=1$. (c) $f(x)=(x-1)(x-5)$.

2. A musician released a new song on a streaming service. A streaming service is an online entertainment source that allows users to play music on their computers and mobile devices.

Several months later, the musician began using an app (at time $t = 0$) that counts the total number of plays for the song since its release. A "play" is a single stream of the song on the streaming service. The table gives the total number of plays, in thousands, for selected times t months after the musician began using the app. At $t = 0$, the total number of plays was 25 thousand. At $t = 2$, the total number of plays was 30 thousand. At $t = 4$, the total number of plays was 34 thousand.

Months after the musician began using the app	0	2	4
Total number of plays for the song since its release (thousands)	25	30	34

The total number of plays, in thousands, for the song since its release can be modeled by the function D given by $D(t) = at^2 + bt + c$, where $D(t)$ is the total number of plays, in thousands, for the song since its release, and t is the number of months after the musician began using the app.

A.

i. Use the given data to write three equations that can be used to find the values for constants a, b, and c in the expression for $D(t)$.

ii. Find the values for a, b, and c as decimal approximations.

B.

i. Use the given data to find the average rate of change of the total number of plays for the song, in thousands per month, from $t = 0$ to $t = 4$ months. Express your answer as a decimal approximation. Show the computations that lead to your answer.

ii. Use the average rate of change found in part B (i) to estimate the total number of plays for the song, in thousands, for $t = 1.5$ months. Show the work that leads to your answer.

iii. Let A_t represent the estimate of the total number of plays for the song, in thousands, using the average rate of change found in part B (i). For $A_{1.5}$ found in part B (ii), it can be shown that $A_{1.5} < D(1.5)$.
Explain why, in general, $A_t < D(t)$ for all t, where $0 < t < 4$. Your explanation should include a reference to the graph of D and its relationship to A_t.

C. The quadratic function model D has exactly one absolute minimum or one absolute maximum. That minimum or maximum can be used to determine a domain restriction for D.

Based on the context of the problem, explain how that minimum or maximum can be used to determine a boundary for the domain of D.

END OF PART A