

Question 1: Function Concepts
Part A: Graphing calculator required
6 points

The function f is decreasing and is defined for all real numbers. The table gives values for $f(x)$ at selected values of x .

x	-2	-1	0	1	2
$f(x)$	14	7	3.5	1.75	0.875

The function g is given by $g(x) = -0.167x^3 + x^2 - 1.834$.

	Model Solution	Scoring
A	(i) The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(1)$ as a decimal approximation or indicate that it is not defined. Show the work that leads to your answer.	
	(ii) Find the value of $f^{-1}(3.5)$, or indicate that it is not defined.	
	(i) $h(1) = g(f(1)) = g(1.75) = 0.333$	Value Point A1
	(ii) From the table, $f^{-1}(3.5) = 0$.	Value Point A2

General Scoring Notes for Question 1 Parts A, B, and C

- Decimal approximations must be accurate to three places after the decimal point by rounding or truncating. Decimal values of 0 in final digits need not be reported ($2.000 = 2.00 = 2.0 = 2$).
- A **decimal presentation error** occurs when a response is complete and correct, but the answer is reported to fewer digits than required.
- The first decimal presentation error in Question 1 does not earn the point. For each additional part of Question 1 that requires a decimal approximation and contains a decimal presentation error, the response is eligible to earn the point.

Scoring Notes for Part A

- Point A1** is earned for a correct decimal approximation of 0.333 with supporting work of “ $f(1)$ ” OR “ $g(1.75)$ ” OR “1.75.”
- Point A2** does not require supporting work. **Point A2** is earned with a response of “0.”

Partial Credit for Part A

A response that **does not** earn either **Point A1** or **Point A2** is eligible for **partial credit** in part A if the response has one criteria from the first column AND one criteria from the second column.

Partial credit response is scored **0** for **Point A1** and **1** for **Point A2**.

First Column	Second Column
Correct value in part A (i) without supporting work.	$f^{-1}(3.5)$ exists in part A (ii) without giving specific value.
Correct value in part A (i) that is not expressed as a decimal approximation.	
Correct value in part A (i) with a decimal presentation error.	

- B**
- (i) Find all values of x , as decimal approximations, for which $g(x) = 0$, or indicate that there are no such values.
- (ii) Determine the end behavior of g as x increases without bound. Express your answer using the mathematical notation of a limit.

(i) $g(x) = 0 \Rightarrow -0.167x^3 + x^2 - 1.834 = 0$ $x = -1.233, x = 1.578, x = 5.643$	Values Point B1
(ii) As x increases without bound, the output values of g decrease without bound. Therefore, $\lim_{x \rightarrow \infty} g(x) = -\infty$.	End behavior with limit notation Point B2

Scoring Notes for Part B

- Point B1** does not require supporting work. **Point B1** is earned with the three correct decimal approximations $-1.233, 1.578$, and 5.643 . The use of “ $x =$ ” is not required.
- Point B2** requires a correct limit statement with four components: “lim”, “ $x \rightarrow \infty$ ”, the function g , and $-\infty$. Examples that earn **Point B2** include:
 - $\lim_{x \rightarrow \infty} g(x) = -\infty$ OR $\lim_{x \rightarrow \infty} g = -\infty$
 - $\lim_{x \rightarrow \infty} g(x) \rightarrow -\infty$ OR $\lim_{x \rightarrow \infty} g \rightarrow -\infty$
 - $\lim_{x \rightarrow \infty} g(x) \quad -\infty$ OR $\lim_{x \rightarrow \infty} g \quad -\infty$
- If the response includes an additional, complete limit statement (e.g., $\lim_{x \rightarrow -\infty} g(x) = \infty$), the limit statement must be correct.

Partial Credit for Part B

A response that **does not** earn either **Point B1** or **Point B2** is eligible for **partial credit** in part B if the response has one criteria from the first column AND one criteria from the second column.

Partial credit response is scored **1** for **Point B1** and **0** for **Point B2**.

First Column	Second Column
Three correct values in part B (i) with values that are not expressed as decimal approximations	Correct end behavior statement in part B (ii) without use of limit notation
Three correct values in part B (i) with a decimal presentation error	Correct end behavior statement in part B (ii) with incorrect limit notation
Only one correct value in part B (i) with no incorrect values included (may have a decimal presentation error)	Correct limit statement in part B (ii) that is missing “ $x \rightarrow \infty$ ”
Only two correct values in part B (i) with no incorrect values included (may have a decimal presentation error)	

- C (i) Based on the table, which of the following function types best models function f : linear, quadratic, exponential, or logarithmic?
- (ii) Give a reason for your answer in part C (i) based on the relationship between the change in the output values of f and the change in the input values of f . Refer to the values in the table in your reasoning.

(i) An exponential function best models f .	Answer	Point C1
(ii) In this case, the input-value intervals all have length 1. Examining the ratios of successive output values gives $\frac{f(-1)}{f(-2)} = \frac{f(0)}{f(-1)} = \frac{f(1)}{f(0)} = \frac{f(2)}{f(1)} = 0.5$ Because the successive output values over equal-length input-value intervals are proportional, an exponential model is best.	Reasoning	Point C2

Scoring Notes for Part C

- **Point C1** is earned for a correct function model with no incorrect function models listed. A response of “exponential” earns **Point C1**.
- Both **Point C1** and **Point C2** may be earned in part C (ii) provided there is no incorrect response in part C (i).
- **Point C2** requires an implicit or explicit reference to output values and input values from the table as support for the reason. For example, $\frac{f(n+1)}{f(n)} = 0.5$ OR “successive output values are decreasing proportionately by a factor of 0.5.” The reasoning must demonstrate that the ratio of 0.5 applies to more than one pair of successive output values.
- A reason that references “exponential regression,” “ r values,” OR “ r^2 values” is not sufficient to earn **Point C2**.
- Special case: A response that indicates that f is best modeled by a **linear, quadratic, or logarithmic** function in part C (i) without a reason in part C (i) combined with a response in part C (ii) that provides both the correct answer and a correct reason is scored **0** for **Point C1** and **1** for **Point C2**.