

2 Sequences and exponential functions

– Part 1

Name: _____ Class: _____ Date: _____

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)
sequence	数列	_____
arithmetic sequence	se- 等差数列	_____
common difference	公差	_____
geometric sequence	se- 等比数列	_____
common ratio	公比	_____
exponential function	指数函数	_____
exponential growth	指数增长	_____
exponential decay	指数衰减	_____

Recall

You have seen number patterns and powers before:

- Sequences like 2, 4, 6, 8, ... (adding) or 2, 4, 8, 16, ... (doubling).
- Powers like $2^3 = 8$.
- Some things grow faster and faster – like money earning interest.

This sheet lines up sequences and exponential growth. Each idea has a worked example.

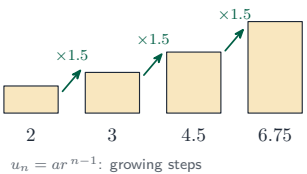
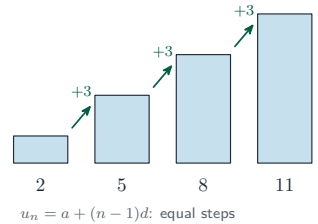
New ideas

■ 1 Two kinds of sequence

A **sequence** 数列 is an ordered list of numbers. Two important types:

arithmetic: add d each step

geometric: multiply by r each step



- an **arithmetic sequence** 等差数列 **adds** a fixed **common difference** 公差 d each step,
- a **geometric sequence** 等比数列 **multiplies** by a fixed **common ratio** 公比 r each step.

Worked example. Arithmetic with $a_0 = 3$, $d = 5$: the terms are 3, 8, 13, 18, ...
Geometric with $g_0 = 2$, $r = 3$: 2, 6, 18, 54, ...

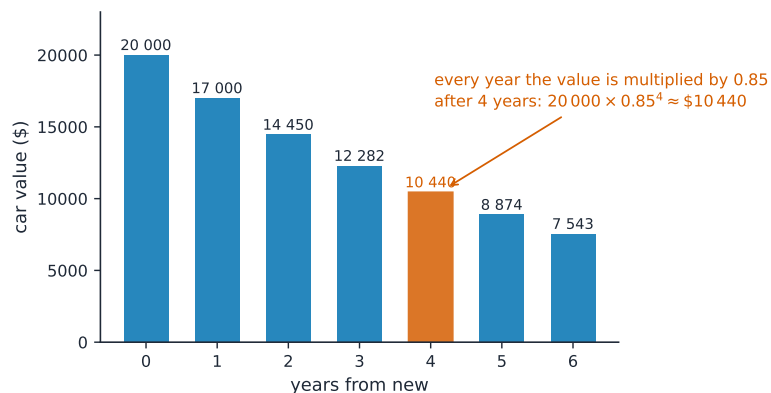
■ 2 Finding a term

For arithmetic, $a_n = a_0 + dn$; for geometric, $g_n = g_0 r^n$.

Worked example. $a_4 = 3 + 5(4) = 23$; $g_4 = 2 \cdot 3^4 = 162$.

■ 3 Exponential functions

An **exponential function** 指数函数 is $f(x) = ab^x$ – a starting value a multiplied by the **base** b repeatedly.



- $b > 1$ gives **exponential growth** 指数增长,
- $0 < b < 1$ gives **exponential decay** 指数衰减.

■ 4 Adding versus multiplying

A linear function **adds** the same amount each step; an exponential function **multiplies** by the same factor. That is why an exponential always overtakes a linear one in the end.

Watch out: exponential growth means multiplying by the same factor each step, so it starts slow but soon shoots up far past any straight line – that is the power of “compounding”.

Practice

Try these in order. The methods are all above.

2.1 State the difference between an arithmetic and a geometric sequence. [2]

2.2 An arithmetic sequence has $a_0 = 4$ and $d = 3$. Find a_5 . [2]

2.3 A geometric sequence has $g_0 = 5$ and $r = 2$. Find g_3 . [2]

2.4 For $f(x) = 3 \cdot 2^x$, find $f(4)$. [2]

2.5 State whether $f(x) = 100 \cdot (0.5)^x$ shows growth or decay, and why. [2]
