

1 Polynomials and rational functions

– Part 2

Name: _____ Class: _____ Date: _____

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)
polynomial	多项式	_____
degree	次数	_____
zero	零点	_____
end behavior	末端行为	_____
rational function	有理函数	_____
vertical asymptote	垂直渐近线	_____
horizontal asymptote	水平渐近线	_____

Recall

In Part A you met functions and their rates of change. Now two important families:

- **Polynomials** like $x^3 - 2x + 1$ – sums of powers of x .
- **Rational functions** – one polynomial divided by another.

Each idea has a worked example before the Practice.

New ideas

■ 1 Polynomials

A **polynomial** 多项式 is a sum of powers of x , like $p(x) = 2x^3 - 5x + 1$. Its highest power is the **degree** 次数, and the term with that power is the **leading term**.

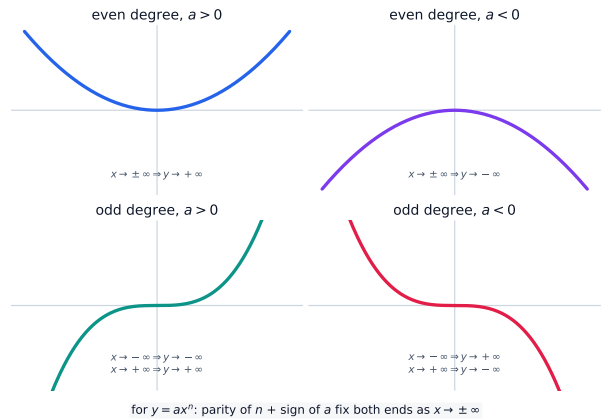
■ 2 Zeros

A **zero** 零点 (or root) of p is an input where $p(x) = 0$. Each real zero gives an **x -intercept** where the graph crosses the x -axis.

Worked example. $p(x) = (x - 2)(x + 3)$ has zeros at $x = 2$ and $x = -3$, so its graph crosses the x -axis at those two points.

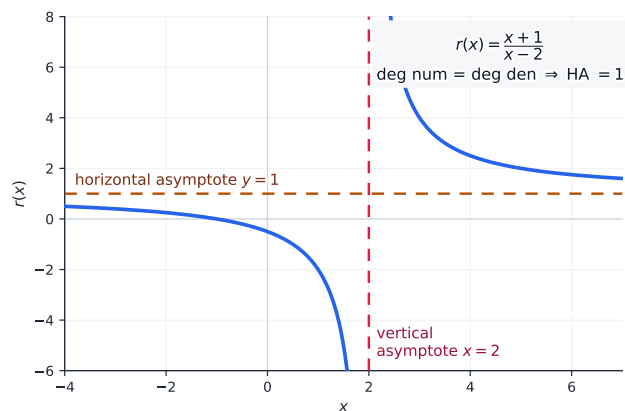
■ 3 End behavior

The **end behavior** 末端行为 – where the graph heads as $x \rightarrow \pm\infty$ – is decided entirely by the leading term.



■ 4 Rational functions and asymptotes

A **rational function** 有理函数 is one polynomial over another. A **vertical asymptote** 垂直渐近线 appears where the denominator is zero (and the top is not), and a **horizontal asymptote** 水平渐近线 describes where the graph levels off far out.



Worked example. For $r(x) = \frac{1}{x-2}$, the denominator is zero at $x = 2$, so there is a vertical asymptote there; far out the value tends to 0, so $y = 0$ is a horizontal asymptote.

Watch out: a graph can **cross** a horizontal asymptote but never crosses a **vertical** one – the function shoots off to $\pm\infty$ as it approaches a vertical asymptote.

Practice

Try these in order. The methods are all above.

2.1 State the degree of $p(x) = 4x^3 - 2x^2 + 7$. [1]

2.2 Find the zeros of $p(x) = (x-1)(x+4)$. [2]

2.3 State where the graph of $p(x) = (x-1)(x+4)$ crosses the x -axis. [1]

2.4 For $r(x) = \frac{1}{x-3}$, state the vertical asymptote. [1]

2.5 For $r(x) = \frac{5}{x+1}$, state the horizontal asymptote as $x \rightarrow \pm\infty$. [1]