1 Polynomials and rational functions – Part 2

Name: _____ Class: _____ Date: _____

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)		
polynomial	多项式			
degree	次数			
zero	零点			
end behavior	末端行为			
rational function	有理函数			
vertical asymptote	垂直渐近线			
horizontal asymptote	水平渐近线			

Recall

In Part A you met functions and their rates of change. Now two important families:

- **Polynomials** like $x^3 - 2x + 1$ –sums of powers of x.
- **Rational functions** –one polynomial divided by another.

Each idea has a worked example before the Practice.

New ideas

■ 1 Polynomials

A **polynomial** 多项式 is a sum of powers of x, like $p(x) = 2x^3 - 5x + 1$. Its highest power is the **degree** 次数, and the term with that power is the **leading term**.

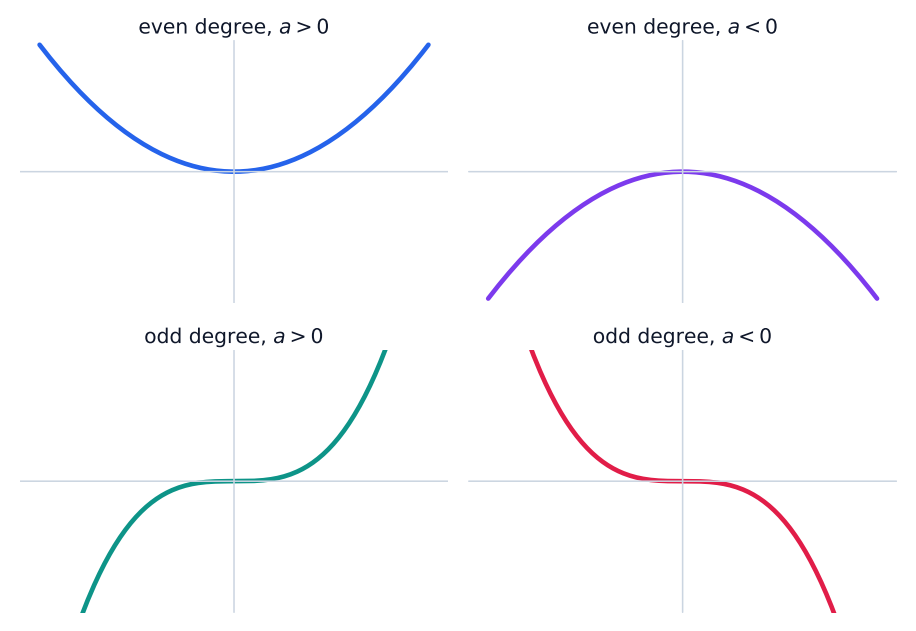
2 Zeros

A **zero** 零点 (or root) of p is an input where $p(x) = 0$. Each real zero gives an **x-intercept** where the graph crosses the x-axis.

Worked example. $p(x) = (x - 2)(x + 3)$ has zeros at $x = 2$ and $x = -3$, so its graph crosses the x-axis at those two points.

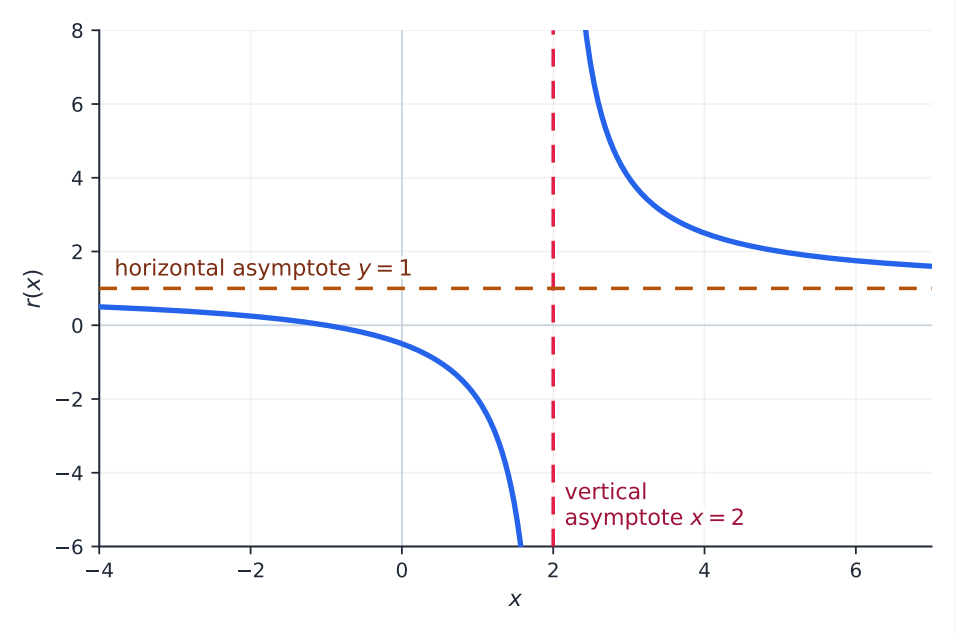
3 End behavior

The **end behavior** 末端行为–where the graph heads as $x \to \pm\infty$ –is decided entirely by the leading term.



4 Rational functions and asymptotes

A **rational function** 有理函数 is one polynomial over another. A **vertical asymptote** 垂直渐近线 appears where the denominator is zero (and the top is not), and a **horizontal asymptote** 水平渐近线 describes where the graph levels off far out.



Worked example. For $r(x) = \dfrac{1}{x-2}$, the denominator is zero at $x = 2$, so there is a vertical asymptote there; far out the value tends to 0, so $y = 0$ is a horizontal asymptote.

Watch out: a graph can **cross** a horizontal asymptote but never crosses a **vertical** one –the function shoots off to $\pm\infty$ as it approaches a vertical asymptote.

Practice

Try these in order. The methods are all above.

2.1 State the degree of $p(x) = 4x^3 - 2x^2 + 7$. [1]

2.2 Find the zeros of $p(x) = (x-1)(x+4)$. [2]

2.3 State where the graph of $p(x) = (x-1)(x+4)$ crosses the x-axis. [1]

2.4 For $r(x) = \dfrac{1}{x-3}$, state the vertical asymptote. [1]

2.5 For $r(x) = \dfrac{5}{x+1}$, state the horizontal asymptote as $x \to \pm\infty$. [1]
