

1 Functions and rates of change – Part 1

Name: _____ Class: _____ Date: _____

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)
function	函数	_____
domain	定义域	_____
range	值域	_____
increasing	递增	_____
decreasing	递减	_____
average rate of change	平均变化率	_____
secant line	割线	_____
linear	线性	_____
quadratic	二次	_____

Recall

You have already worked with functions and graphs:

- A function turns each input into one output, like $f(x) = x^2$.
- You have found the gradient (slope) of a straight line.
- You know a quadratic makes a U-shaped parabola.

This sheet lines up how functions change. Each idea has a worked example.

New ideas

■ 1 Functions, domain, and range

A **function** 函数 gives each input exactly one output. The allowed inputs are the **domain** 定义域; the outputs are the **range** 值域.

■ 2 Increasing and decreasing

A function is **increasing** 递增 where a bigger input gives a bigger output, and **decreasing** 递减 where a bigger input gives a smaller output.

■ 3 Average rate of change

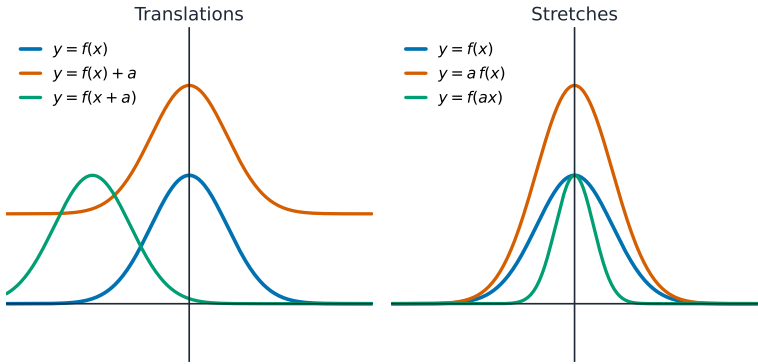
The **average rate of change** 平均变化率 over an interval $[a, b]$ is the change in output over the change in input:

$$\text{avg rate} = \frac{f(b) - f(a)}{b - a}.$$

This is just the gradient of the straight line (the **secant line** 割线) joining the two points.

Worked example. For $f(x) = x^2$ over $[1, 4]$: $\frac{f(4) - f(1)}{4 - 1} = \frac{16 - 1}{3} = 5$.

■ 4 Linear versus quadratic



For a **linear** 线性 function the average rate of change is **constant**; for a **quadratic** 二次 function it **changes** from interval to interval.

Watch out: the average rate of change is the slope of the line between two points – so for a curve it depends on **which** two points you pick. Only a straight line has the same rate everywhere.

Practice

Try these in order. The methods are all above.

2.1 State what the domain and the range of a function are. [2]

2.2 For $f(x) = 3x + 1$, find the average rate of change over $[0, 4]$. [2]

2.3 For $f(x) = x^2$, find the average rate of change over $[2, 5]$. [3]

2.4 State whether $f(x) = x^2$ is increasing or decreasing for $x > 0$. [1]

2.5 Explain why a linear function has the same average rate of change over every interval. [2]
