

4.11 The Inverse and Determinant of a Matrix

Name: _____ Class: _____ Date: _____ Total: 9 marks

KEY WORDS determinant 行列式 • inverse 逆矩阵 • matrix 矩阵

Objective

Build the skills to answer exam questions on **the inverse and determinant of a matrix**.

You must be able to:

- compute the **determinant** 行列式 of a 2×2 matrix
- decide whether a matrix has an **inverse** 逆矩阵

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Determinant and inverse

For $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the determinant is $\det(A) = ad - bc$.

A has an inverse if and only if $\det(A) \neq 0$, and then

$$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

■ Example

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}: \det = 2(1) - 1(1) = 1, \text{ so } A^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}.$$

2 Practice

2.1 State the determinant of $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$. [1]

2.2 State when a matrix has an inverse. [1]

2.3 Find the determinant of $\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$. [2]

3 Exam-style questions

3.1 $\det \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ equals [1]

- | | |
|---|---|
| A | B |
| C | D |

A $ad + bc$
B $ad - bc$
C $ab - cd$
D $a + d$

3.2 A 2×2 matrix has an inverse if and only if [1]

- | | |
|---|---|
| A | B |
| C | D |

A it is square
B $\det \neq 0$
C it is diagonal
D all entries are positive

3.3 $A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$.

- (a) Find $\det(A)$. [1]
(b) State whether A is invertible. [1]
(c) Find A^{-1} . [1]