

1.9 Rational Functions and Vertical Asymptotes

Name: _____ Class: _____ Date: _____ Total: 8 marks

KEY WORDS vertical asymptote 垂直渐近线 • function 函数 • zero 零点
• rational function 有理函数 • leading term 首项

Objective

Build the skills to answer exam questions on **rational functions and vertical asymptotes**.

You must be able to:

- identify a **vertical asymptote** 垂直渐近线 at $x = a$
- describe the behaviour of $r(x)$ near a vertical asymptote

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Vertical asymptotes

A rational function has a **vertical asymptote** at $x = a$ if a is a zero of the **denominator** but **not** of the numerator (or has higher multiplicity in the denominator).

■ Behaviour near it

Near $x = a$, the denominator is close to 0, so the values **increase or decrease without bound**: $\lim_{x \rightarrow a^+} r(x) = \pm\infty$.

For $r(x) = \frac{1}{x-3}$, the vertical asymptote is $x = 3$.

2 Practice

2.1 State the condition for a vertical asymptote at $x = a$. [1]

2.2 For $r(x) = \frac{1}{x-3}$, state the vertical asymptote. [1]

2.3 State what happens to $r(x)$ near a vertical asymptote. [1]

3 Exam-style questions

3.1 A rational function has a vertical asymptote where the _____ is zero (but not the numerator). [1]

- | | |
|---|---|
| A | B |
| C | D |
- A numerator
 - B denominator
 - C leading term
 - D degree

3.2 Near a vertical asymptote, the function values [1]

- | | |
|---|---|
| A | B |
| C | D |
- A approach 0
 - B increase or decrease without bound
 - C stay constant
 - D equal the mean

3.3 $r(x) = \frac{x+2}{x-5}$.

- (a) State the vertical asymptote. [1]
- (b) State the zero. [1]
- (c) State the domain restriction. [1]