

1.10 Rational Functions and Holes

Name: _____ Class: _____ Date: _____ **Total: 9 marks**

KEY WORDS hole 空心点 • function 函数 • rational function 有理函数
• vertical asymptote 垂直渐近线 • leading term 首项

Objective

Build the skills to answer exam questions on **rational functions and holes**.

You must be able to:

- identify a **hole** 空心点 where a factor cancels from numerator and denominator
- find the location (c, L) of a hole

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Holes

A rational function has a **hole** at $x = c$ when a factor $(x - c)$ **cancels** — the numerator's multiplicity of c is at least the denominator's.

■ Location

The hole is at (c, L) , where $L = \lim_{x \rightarrow c} r(x)$ — evaluate the **simplified** function at c .

■ Example

$r(x) = \frac{(x-2)(x+1)}{x-2}$ simplifies to $x+1$ (for $x \neq 2$), so there is a hole at $(2, 3)$.

2 Practice

2.1 State when a rational function has a hole. [1]

2.2 For $r(x) = \frac{(x-2)(x+1)}{x-2}$, state the location of the hole. [2]

2.3 State the difference between a hole and a vertical asymptote. [1]

3 Exam-style questions

3.1 A rational function has a hole where a factor [1]

- | | |
|---|---|
| A | B |
| C | D |
- A appears only in the denominator
B cancels from numerator and denominator
C is the leading term
D is constant

3.2 The x -value of a hole is [1]

- | | |
|---|---|
| A | B |
| C | D |
- A always 0
B where a common factor is zero
C a vertical asymptote
D undefined for all x

3.3 $r(x) = \frac{(x-3)(x+4)}{x-3}$.

- (a) Simplify $r(x)$ (for $x \neq 3$). [1]
(b) State the x -coordinate of the hole. [1]
(c) Find the y -coordinate of the hole. [1]