

# 4.7 Parametrization of Implicitly Defined Functions

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_ Total: 9 marks

KEY WORDS parametric 参数 • implicit 隐式

## Objective

Build the skills to answer exam questions on **parametrization of implicitly defined functions**.

You must be able to:

- convert an implicit curve to **parametric** 参数 form
- use  $\cos^2 t + \sin^2 t = 1$  to parametrize a circle

## 1 Worked examples

Study these first. Each one shows the method for a question type used later.

### ■ Parametrizing an implicit curve

Rewrite an implicit relation as  $x = f(t)$ ,  $y = g(t)$ . The circle  $x^2 + y^2 = r^2$  becomes

$$x = r \cos t, \quad y = r \sin t,$$

which works because  $\cos^2 t + \sin^2 t = 1$ .

### ■ Example

$$x^2 + y^2 = 9 \Rightarrow x = 3 \cos t, y = 3 \sin t.$$

## 2 Practice

2.1 Parametrize  $x^2 + y^2 = 16$ . [2]

2.2 State the identity used to parametrize a circle. [1]

2.3 Verify that  $x = 2 \cos t$ ,  $y = 2 \sin t$  satisfies  $x^2 + y^2 = 4$ . [1]

## 3 Exam-style questions

3.1 The circle  $x^2 + y^2 = 25$  parametrizes as [1]

- A  $x = 5t$ ,  $y = 5t$
- B  $x = 5 \cos t$ ,  $y = 5 \sin t$
- C  $x = t^2$ ,  $y = t^2$
- D  $x = 25t$ ,  $y = 25t$

3.2 Parametrizing a circle uses the identity [1]

- A  $\sin t + \cos t = 1$
- B  $\cos^2 t + \sin^2 t = 1$
- C  $\tan t = \sin t / \cos t$
- D  $\sec t = 1 / \cos t$

3.3  $x^2 + y^2 = 36$ .

(a) State  $r$ . [1]

(b) Write  $x(t)$ . [1]

(c) Write  $y(t)$ . [1]

## Solutions

---

**2.1**  $x = 4 \cos t$ ,  $y = 4 \sin t$ .

**2.2**  $\cos^2 t + \sin^2 t = 1$ .

**2.3**  $x^2 + y^2 = 4 \cos^2 t + 4 \sin^2 t = 4(\cos^2 t + \sin^2 t) = 4$ .

**3.1** B.

**3.2** B.

**3.3** (a) 6. (b)  $x = 6 \cos t$ . (c)  $y = 6 \sin t$ .