

4.5 Implicitly Defined Functions

Name: _____ Class: _____ Date: _____ **Total: 9 marks**

KEY WORDS implicit 隐式 • implicitly defined 隐式定义

Objective

Build the skills to answer exam questions on **implicitly defined functions**.

You must be able to:

- recognise an **implicitly defined** 隐式定义 relation such as $x^2 + y^2 = 25$
- solve it for y and decide whether it is a function

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ **Implicit relations**

An implicit equation relates x and y without solving for y — for example $x^2 + y^2 = 25$ (a circle). It may fail the vertical-line test, so it need not be a function.

■ **Example**

$x^2 + y^2 = 25$ is a circle of radius 5; solving gives $y = \pm\sqrt{25 - x^2}$.

2 Practice

2.1 State what “implicitly defined” means. [1]

2.2 State the shape of $x^2 + y^2 = 9$. [1]

2.3 Solve $x^2 + y^2 = 25$ for y . [2]

3 Exam-style questions

3.1 An implicitly defined relation is one [1]

- **A** solved for y
- **B** written as an equation in x and y
- **C** always a function
- **D** always linear

3.2 The graph of $x^2 + y^2 = 16$ is [1]

- **A** a line
- **B** a circle of radius 4
- **C** a parabola
- **D** two points

3.3 $x^2 + y^2 = 25$.

(a) State the radius. [1]

(b) Solve for y . [1]

(c) State whether it is a function. [1]

Solutions

2.1 given by an equation in x and y that is not solved for y .

2.2 a circle of radius 3 centred at the origin.

2.3 $y = \pm\sqrt{25 - x^2}$.

3.1 B.

3.2 B.

3.3 (a) 5. (b) $y = \pm\sqrt{25 - x^2}$. (c) no (it fails the vertical-line test).