

4.3 Parametric Functions and Rates of Change

Name: _____ Class: _____ Date: _____ Total: 9 marks

KEY WORDS parametric function 参数函数

Objective

Build the skills to answer exam questions on **parametric functions and rates of change**.

You must be able to:

- find the average rate of change of x and of y with respect to t
- find the slope $\frac{dy}{dx}$ of a parametric curve

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Parametric rates of change

- rate of change of x : $\frac{\Delta x}{\Delta t}$; of y : $\frac{\Delta y}{\Delta t}$;
- the slope of the curve: $\frac{dy}{dx} = \frac{\Delta y / \Delta t}{\Delta x / \Delta t}$.

■ Example

$x = 2t$, $y = t^2$ over $[0, 1]$: $\Delta x = 2$, $\Delta y = 1$, slope $= \frac{1}{2}$.

2 Practice

2.1 State the average rate of change of x on $[t_1, t_2]$. [1]

2.2 State the slope $\frac{dy}{dx}$ in parametric terms. [1]

2.3 For $x = t$, $y = 2t$ over $[0, 3]$, find the average rates of change of x and y . [2]

3 Exam-style questions

3.1 The slope of a parametric curve is [1]

- A $\Delta x / \Delta y$
- B $\frac{\Delta y / \Delta t}{\Delta x / \Delta t}$
- C $\Delta t / \Delta y$
- D $\Delta x \cdot \Delta y$

3.2 The average rate of change of x over $[a, b]$ is [1]

- A $\frac{x(b) - x(a)}{b - a}$
- B $x(b) \cdot x(a)$
- C $b - a$
- D $x(b) + x(a)$

3.3 $x = 3t$, $y = t$.

(a) Find Δx over $[0, 2]$. [1]

(b) Find Δy over $[0, 2]$. [1]

(c) Find the slope $\frac{dy}{dx}$. [1]

Solutions

2.1 $\frac{x(t_2) - x(t_1)}{t_2 - t_1}.$

2.2 $\frac{dy}{dx} = \frac{\Delta y / \Delta t}{\Delta x / \Delta t}.$

2.3 $x: \frac{3-0}{3} = 1; y: \frac{6-0}{3} = 2.$

3.1 B.

3.2 A.

3.3 (a) $\Delta x = 6.$ (b) $\Delta y = 2.$ (c) $\frac{2}{6} = \frac{1}{3}.$