

3.12 Equivalent Representations of Trigonometric Functions

Name: _____ Class: _____ Date: _____ Total: 9 marks

KEY WORDS pythagorean identity 勾股恒等式

Objective

Build the skills to answer exam questions on **equivalent representations of trigonometric functions**.

You must be able to:

- apply the **Pythagorean identity** 勾股恒等式 $\sin^2 x + \cos^2 x = 1$
- use the even/odd symmetry of sine and cosine

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Key identities

- **Pythagorean:** $\sin^2 x + \cos^2 x = 1$, so $\cos^2 x = 1 - \sin^2 x$;
- **even/odd:** $\cos(-x) = \cos x$ (even), $\sin(-x) = -\sin x$ (odd).

■ Example

If $\sin x = \frac{3}{5}$, then $\cos^2 x = 1 - \frac{9}{25} = \frac{16}{25}$, so $\cos x = \pm \frac{4}{5}$.

2 Practice

2.1 State the Pythagorean identity. [1]

2.2 Rewrite $1 - \cos^2 x$. [1]

2.3 State $\sin(-x)$ in terms of $\sin x$. [2]

3 Exam-style questions

3.1 The Pythagorean identity is [1]

- A $\sin x + \cos x = 1$
- B $\sin^2 x + \cos^2 x = 1$
- C $\sin^2 x - \cos^2 x = 1$
- D $\sin x \cdot \cos x = 1$

3.2 $\cos(-x)$ equals [1]

- A $-\cos x$
- B $\cos x$
- C $\sin x$
- D $-\sin x$

3.3 Given $\sin x = \frac{3}{5}$.

(a) State $\sin^2 x$. [1]

(b) Use the identity to find $\cos^2 x$. [1]

(c) State a possible value of $\cos x$. [1]

Solutions

2.1 $\sin^2 x + \cos^2 x = 1.$

2.2 $\sin^2 x.$

2.3 $\sin(-x) = -\sin x.$

3.1 B.

3.2 B.

3.3 (a) $\frac{9}{25}.$ (b) $1 - \frac{9}{25} = \frac{16}{25}.$ (c) $\frac{4}{5}$ (or $-\frac{4}{5}$).