

2.12 Logarithmic Function Manipulation

Name: _____ Class: _____ Date: _____ **Total: 9 marks**

KEY WORDS laws of logarithms 对数律 • change-of-base 换底 • logarithm 对数 • base 底数
• logarithmic function 对数函数

Objective

Build the skills to answer exam questions on **logarithmic function manipulation**.

You must be able to:

- apply the **laws of logarithms** 对数律 (product, quotient, power)
- use the **change-of-base** 换底 formula

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ **Laws of logarithms**

$$\log_b(xy) = \log_b x + \log_b y, \quad \log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y, \quad \log_b(x^n) = n \log_b x.$$

■ **Change of base**

$$\log_b(x) = \frac{\ln x}{\ln b}.$$

■ **Example**

$$\log_2(8x) = \log_2 8 + \log_2 x = 3 + \log_2 x.$$

2 Practice

2.1 Expand $\log(xy)$. [1]

2.2 Write $\log(x^3)$ using the power rule. [1]

2.3 Combine $\log 6 + \log 2$ into a single logarithm. [2]

3 Exam-style questions

3.1 $\log_b(xy)$ equals [1]

- **A** $\log_b x \cdot \log_b y$
- **B** $\log_b x + \log_b y$
- **C** $\log_b x - \log_b y$
- **D** $\log_b(x + y)$

3.2 $\log_b(x^n)$ equals [1]

- **A** $n + \log_b x$
- **B** $n \log_b x$
- **C** $(\log_b x)^n$
- **D** $\log_b(nx)$

3.3 Expand $\log_2(8x)$.

(a) Apply the product rule. [1]

(b) Evaluate $\log_2 8$. [1]

(c) Write the final expression. [1]

Solutions

2.1 $\log x + \log y.$

2.2 $3 \log x.$

2.3 $\log(6 \cdot 2) = \log 12.$

3.1 B.

3.2 B.

3.3 (a) $\log_2 8 + \log_2 x.$ (b) 3. (c) $3 + \log_2 x.$