

1.2 Rates of Change

Name: _____ Class: _____ Date: _____ **Total: 9 marks**

KEY WORDS average rate of change 平均变化率 • degree 次数 • domain 定义域 • range 值域

Objective

Build the skills to answer exam questions on **rates of change**.

You must be able to:

- calculate the **average rate of change** 平均变化率 over an interval
- describe the **rate of change at a point** 某点变化率

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Average rate of change

Over an interval $[a, b]$:

$$\text{AROC} = \frac{f(b) - f(a)}{b - a},$$

the ratio of the change in output to the change in input.

■ Rate of change at a point

The rate of change **at a point** describes how fast the output would change there; it can be **approximated** by average rates over small intervals containing the point.

For $f(1) = 3$, $f(4) = 12$: $\text{AROC} = \frac{12 - 3}{4 - 1} = 3$.

2 Practice

2.1 Write the formula for the average rate of change over $[a, b]$. [1]

2.2 If $f(2) = 5$ and $f(6) = 17$, find the average rate of change over $[2, 6]$. [2]

2.3 State what the rate of change at a point describes. [1]

3 Exam-style questions

3.1 The average rate of change of f over $[a, b]$ is [1]

- **A** $f(b) - f(a)$
- **B** $\frac{f(b) - f(a)}{b - a}$
- **C** $b - a$
- **D** $f(b) + f(a)$

3.2 The rate of change at a point can be approximated using [1]

- **A** the range
- **B** average rates over small intervals
- **C** the domain
- **D** the degree

3.3 $f(0) = 2$ and $f(5) = 22$, where f is distance (m) and x is time (s).

(a) Find the average rate of change over $[0, 5]$. [2]

(b) State its units interpretation. [1]

Solutions

2.1 $\frac{f(b) - f(a)}{b - a}.$

2.2 $\frac{17 - 5}{6 - 2} = \frac{12}{4} = 3.$

2.3 how fast the output would change at that input value.

3.1 B.

3.2 B.

3.3 (a) $\frac{22 - 2}{5 - 0} = 4.$ (b) 4 metres per second (average speed).