Exercise walk-through

Parametrization of Implicitly Defined Functions ▪ 4.7

eXercise

You must be able to

- convert an implicit curve to **parametric 参数** form
- use $\cos^2 t + \sin^2 t = 1$ to parametrize a circle

Method — Parametrizing an implicit curve

Rewrite an implicit relation as $x = f(t)$, $y = g(t)$. The circle $x^2 + y^2 = r^2$ becomes

$$x = r\cos t, \quad y = r\sin t,$$

which works because $\cos^2 t + \sin^2 t = 1$.

Method — Example

$$x^2 + y^2 = 9 \;\Rightarrow\; x = 3\cos t,\; y = 3\sin t.$$

Practice 2.1 ▪ 2 marks

Parametrize $x^2 + y^2 = 16$.

Solution 2.1

Worked steps

$x = 4\cos t,\ y = 4\sin t$ M1 A1.

Practice 2.2 ▪ 1 mark

State the identity used to parametrize a circle.

Solution 2.2

Worked steps

$\cos^2 t + \sin^2 t = 1$ **B1**.

Practice 2.3 ▪ 1 mark

Verify that $x = 2\cos t$, $y = 2\sin t$ satisfies $x^2 + y^2 = 4$.

Solution 2.3

Worked steps

$x^2 + y^2 = 4\cos^2 t + 4\sin^2 t = 4(\cos^2 t + \sin^2 t) = 4$ **B1**.

Exam-style 3.1 ▪ 1 mark

The circle $x^2 + y^2 = 25$ parametrizes as

A $x = 5t,\ y = 5t$

B $x = 5\cos t,\ y = 5\sin t$

C $x = t^2,\ y = t^2$

D $x = 25t,\ y = 25t$

Solution 3.1

A $x = 5t,\ y = 5t$

B $x = 5\cos t,\ y = 5\sin t$ ✓

C $x = t^2,\ y = t^2$

D $x = 25t,\ y = 25t$

Worked steps

B.

Exam-style 3.2 ▪ 1 mark

Parametrizing a circle uses the identity

A $\sin t + \cos t = 1$

B $\cos^2 t + \sin^2 t = 1$

C $\tan t = \sin t / \cos t$

D $\sec t = 1 / \cos t$

Solution 3.2

Method: Parametrizing an implicit curve

- **A** $\sin t + \cos t = 1$
- **B** $\cos^2 t + \sin^2 t = 1$ ✓
- **C** $\tan t = \sin t / \cos t$
- **D** $\sec t = 1 / \cos t$

Worked steps

B.

Exam-style 3.3 ▪ 1 mark

$x^2 + y^2 = 36$.
(a) State r.
(b) Write $x(t)$.
(c) Write $y(t)$.

Solution 3.3

Worked steps

(a) 6 **B1**. (b) $x = 6\cos t$ **B1**. (c) $y = 6\sin t$ **B1**.