

Exercise walk-through

Parametric Functions and Rates of Change ■ 4.3

eXercise

You must be able to

- find the average rate of change of x and of y with respect to t
- find the slope $\frac{dy}{dx}$ of a parametric curve

Method — Parametric rates of change

- rate of change of x : $\frac{\Delta x}{\Delta t}$; of y : $\frac{\Delta y}{\Delta t}$;
- the slope of the curve: $\frac{dy}{dx} = \frac{\Delta y / \Delta t}{\Delta x / \Delta t}$.

Method — Example

$$x = 2t, y = t^2 \text{ over } [0, 1]: \Delta x = 2, \Delta y = 1, \text{ slope} = \frac{1}{2}.$$

Practice 2.1 ■ 1 mark

State the average rate of change of x on $[t_1, t_2]$.

Solution 2.1

Method: Parametric rates of change

Worked steps

$$\frac{x(t_2) - x(t_1)}{t_2 - t_1} \quad \text{B1}.$$

Practice 2.2 ■ 1 mark

State the slope $\frac{dy}{dx}$ in parametric terms.

Solution 2.2

Method: Parametric rates of change

Worked steps

$$\frac{dy}{dx} = \frac{\Delta y / \Delta t}{\Delta x / \Delta t} \quad \text{B1}.$$

Practice 2.3 ■ 2 marks

For $x = t$, $y = 2t$ over $[0, 3]$, find the average rates of change of x and y .

Solution 2.3

Method: Parametric rates of change

Worked steps

$$x: \frac{3-0}{3} = 1; y: \frac{6-0}{3} = 2 \quad \text{B1} \quad \text{B1}.$$

Exam-style 3.1 ■ 1 mark

The slope of a parametric curve is

A $\Delta x / \Delta y$

B $\frac{\Delta y / \Delta t}{\Delta x / \Delta t}$

C $\Delta t / \Delta y$

D $\Delta x \cdot \Delta y$

Solution 3.1

Method: Parametric rates of change

A $\Delta x / \Delta y$

B $\frac{\Delta y / \Delta t}{\Delta x / \Delta t}$



C $\Delta t / \Delta y$

D $\Delta x \cdot \Delta y$

Worked steps

B.

Exam-style 3.2 ■ 1 mark

The average rate of change of x over $[a, b]$ is

A $\frac{x(b) - x(a)}{b - a}$

B $x(b) \cdot x(a)$

C $b - a$

D $x(b) + x(a)$

Solution 3.2

Method: Parametric rates of change

A $\frac{x(b) - x(a)}{b - a}$



B $x(b) \cdot x(a)$

C $b - a$

D $x(b) + x(a)$

Worked steps

A.

Exam-style 3.3 ■ 1 mark

$$x = 3t, y = t.$$

(a) Find Δx over $[0, 2]$.

(b) Find Δy over $[0, 2]$.

(c) Find the slope $\frac{dy}{dx}$.

Solution 3.3

Worked steps

(a) $\Delta x = 6$ **B1**. (b) $\Delta y = 2$ **B1**. (c) $\frac{2}{6} = \frac{1}{3}$ **B1**.