Exercise walk-through

Matrices Modeling Contexts ▪ 4.14

eXercise

You must be able to

- use a **transition matrix 转移矩阵** to step a state vector forward

Method — Transition matrices

A transition matrix models a process that moves between states each step:

$$\text{next state} = (\text{transition matrix}) \times (\text{current state}).$$

Repeated multiplication steps the model forward in time.

Method — Example

State $\begin{bmatrix} \text{city} \\ \text{suburb} \end{bmatrix}$: each year a transition matrix redistributes the population; multiply to get next year's distribution.

Practice 2.1 ▪ 1 mark

State what a transition matrix models.

Solution 2.1

Method: Transition matrices

Worked steps

a process that moves between states each step B1.

Practice 2.2 ▪ 1 mark

State how to get the next state from the current state.

Solution 2.2

Method: Transition matrices

Worked steps

multiply the transition matrix by the current state vector B1.

Practice 2.3 ▪ 1 mark

If the current state is $\begin{bmatrix} 10 \\ 20 \end{bmatrix}$ and the transition is the identity matrix, state the next state.

Solution 2.3

Method: Example

Worked steps

$\begin{bmatrix}10\\20\end{bmatrix}$ (unchanged) B1.

Exam-style 3.1 ▪ 1 mark

A transition matrix models

A a single number

B movement between states over steps

C a static picture

D an angle

Solution 3.1

Method: Transition matrices

A a single number

B movement between states over steps ✓

C a static picture

D an angle

Worked steps

B.

Exam-style 3.2 ▪ 1 mark

The next state equals

A current state minus the matrix

B matrix times current state

C matrix plus current state

D the current state alone

Solution 3.2

A current state minus the matrix

B matrix times current state ✓

C matrix plus current state

D the current state alone

Worked steps

B.

Exam-style 3.3 ▪ 1 mark

Current state $\begin{bmatrix} 100 \\ 0 \end{bmatrix}$; transition $\begin{bmatrix} 0.5 & 0 \\ 0.5 & 1 \end{bmatrix}$ (half of state 1 moves to state 2 each step).

(a) Compute the next state.

(b) State the new value of state 1.

(c) State the new value of state 2.

Solution 3.3

Method: Transition matrices

Worked steps

(a) $\begin{bmatrix} 0.5(100)+0 \\ 0.5(100)+1(0) \end{bmatrix} = \begin{bmatrix} 50 \\ 50 \end{bmatrix}$ B1. (b) 50 B1. (c) 50 B1.