Exercise walk-through

Matrices as Functions ▪ 4.13

eXercise

You must be able to

- view a matrix transformation $T(\vec{v}) = A\vec{v}$ as a **function 函数** on vectors
- relate composition of transformations to the matrix product

Method — Matrices as functions

A matrix transformation $T(\vec{v}) = A\vec{v}$ is a function taking an input vector to an output vector. Applying two transformations in sequence corresponds to **multiplying** their matrices: applying A then B is $B(A\vec{v}) = (BA)\vec{v}$.

Method — Example

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \text{ swaps components: } T\left(\begin{bmatrix} 2 \\ 5 \end{bmatrix}\right) = \begin{bmatrix} 5 \\ 2 \end{bmatrix}.$$

Practice 2.1 ▪ 1 mark

State what the input and output of a matrix transformation are.

Solution 2.1

Method: Matrices as functions

Worked steps

the input is a vector and the output is a vector **B1**.

Practice 2.2 ▪ 1 mark

State what composing two matrix transformations corresponds to.

Solution 2.2

Method: Matrices as functions

Worked steps

multiplying their matrices **B1**.

Practice 2.3 ▪ 2 marks

If $T(\vec{v}) = A\vec{v}$ and $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, find $T\left(\begin{bmatrix} 2 \\ 5 \end{bmatrix}\right)$.

Solution 2.3

Method: Example

Worked steps

$\begin{bmatrix} 5 \\ 2 \end{bmatrix}$ **M1** **A1**.

Exam-style 3.1 ▪ 1 mark

A matrix transformation $T(\vec{v}) = A\vec{v}$ takes

A a number to a number

B a vector to a vector

C a matrix to a scalar

D nothing

Solution 3.1

Method: Matrices as functions

- **A** a number to a number
- **B** a vector to a vector ✓
- **C** a matrix to a scalar
- **D** nothing

Worked steps

B.

Exam-style 3.2 ▪ 1 mark

Applying transformation A then B equals the single matrix

A $A + B$

B BA

C $A - B$

D AB only if diagonal

Solution 3.2

Method: Matrices as functions

A $A + B$

B BA ✓

C $A - B$

D AB only if diagonal

Worked steps

B.

Exam-style 3.3 ▪ 1 mark

$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

(a) Find $A \begin{bmatrix} 2 \\ 5 \end{bmatrix}$.

(b) Find $A \begin{bmatrix} 7 \\ 0 \end{bmatrix}$.

(c) Describe the effect.

Solution 3.3

Method: Example

Worked steps

(a) $\begin{bmatrix} 5 \\ 2 \end{bmatrix}$ **B1**. (b) $\begin{bmatrix} 0 \\ 7 \end{bmatrix}$ **B1**. (c) it swaps the two components (a reflection across $y = x$) **B1**.