Exercise walk-through

Linear Transformations and Matrices ▪ 4.12

eXercise

You must be able to

- apply a matrix to a vector by **matrix-vector multiplication 矩阵向量乘法**
- describe the geometric effect (scaling, identity)

Method — Linear transformations

A 2×2 matrix acts on a vector:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} ax + by \\ cx + dy \end{bmatrix}.$$

This transforms the plane — scaling, rotation, or reflection.

Method — Example

$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$$ — a scaling by 2.

Practice 2.1 ▪ 1 mark

State how a matrix acts on a vector.

Solution 2.1

Method: Linear transformations

Worked steps

by matrix-vector multiplication, giving $\begin{bmatrix} ax + by \\ cx + dy \end{bmatrix}$ B1.

Practice 2.2 ▪ 1 mark

Compute $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \end{bmatrix}$.

Solution 2.2

Worked steps

$\begin{bmatrix} 5 \\ 2 \end{bmatrix}$ **B1**.

Practice 2.3 ▪ 2 marks

Compute $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

Solution 2.3

Worked steps

$$\begin{bmatrix} 2 \\ 3 \end{bmatrix}$$ **M1** **A1**.

Exam-style 3.1 ▪ 1 mark

The identity matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ maps a vector to

A the zero vector

B itself

C its reverse

D twice itself

Solution 3.1

Method: Linear transformations

A the zero vector

B itself ✓

C its reverse

D twice itself

Worked steps

B.

Exam-style 3.2 ▪ 1 mark

$\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ applied to a vector gives a

A rotation

B scaling by 3

C reflection

D shift

Solution 3.2

Method: Linear transformations

A rotation

B scaling by 3 ✓

C reflection

D shift

Worked steps

B.

Exam-style 3.3 ▪ 1 mark

$A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$.

(a) Compute $A \begin{bmatrix} 1 \\ 4 \end{bmatrix}$.

(b) Compute $A \begin{bmatrix} 3 \\ 0 \end{bmatrix}$.

(c) Describe the effect.

Solution 3.3

Worked steps

(a) $\begin{bmatrix} 2 \\ 4 \end{bmatrix}$ **B1**. (b) $\begin{bmatrix} 6 \\ 0 \end{bmatrix}$ **B1**. (c) it stretches the x-component by 2 and leaves y unchanged **B1**.