Exercise walk-through

The Inverse and Determinant of a Matrix ▪ 4.11

eXercise

You must be able to

- compute the **determinant 行列式** of a 2×2 matrix
- decide whether a matrix has an **inverse 逆矩阵**

Method — Determinant and inverse

For $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the determinant is $\det(A) = ad - bc$.

A has an inverse if and only if $\det(A) \neq 0$, and then

$$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

Method — Example

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\text{: } \det = 2(1) - 1(1) = 1\text{, so } A^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}.$$

Practice 2.1 ▪ 1 mark

State the determinant of $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

Solution 2.1

Method: Determinant and inverse

Worked steps

$ad - bc$ B1.

Practice 2.2 ▪ 1 mark

State when a matrix has an inverse.

Solution 2.2

Worked steps

when its determinant is non-zero B1.

Practice 2.3 ▪ 2 marks

Find the determinant of $\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$.

Solution 2.3

Method: Determinant and inverse

Worked steps

$3(4) - 2(1) = 12 - 2 = 10$ M1 A1.

Exam-style 3.1 ▪ 1 mark

$\det \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ equals

A $ad + bc$

B $ad - bc$

C $ab - cd$

D $a + d$

Solution 3.1

A $ad + bc$

B $ad - bc$ ✓

C $ab - cd$

D $a + d$

Worked steps

B.

Exam-style 3.2 ▪ 1 mark

A 2×2 matrix has an inverse if and only if

A it is square

B $\det \neq 0$

C it is diagonal

D all entries are positive

Solution 3.2

Method: Determinant and inverse

A it is square

B $\det \neq 0$ ✓

C it is diagonal

D all entries are positive

Worked steps

B.

Exam-style 3.3 ▪ 1 mark

$A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$.

(a) Find $\det(A)$.

(b) State whether A is invertible.

(c) Find A^{-1}.

Solution 3.3

Worked steps

(a) $2(3) - 0 = 6$ B1. (b) yes, since $\det = 6 \neq 0$ B1. (c) $\begin{bmatrix} 1/2 & 0 \\ 0 & 1/3 \end{bmatrix}$ B1.