Past-paper walk-through

Inverse Trigonometric Functions ▪ 3.9

eXercise

Questions in this set

- 2024 Q4

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 4

2024 ▪ Q4

Directions:
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians. - Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator. - Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms. - For each part of the question, show the work that leads to your answers.

Question 4

(a)(i) The functions g and h are given by

$$g(x) = e^{(x+3)}$$

$$h(x) = \arcsin\left(\frac{x}{2}\right).$$

Solve $g(x) = 10$ for values of x in the domain of g.

(a)(ii) Solve $h(x) = \frac{\pi}{4}$ for values of x in the domain of h.

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Question 4

(b)(i) The functions j and k are given by

$$j(x) = \log_{10}(8x^5) + \log_{10}(2x^2) - 9\log_{10} x$$

$$k(x) = \left(\frac{1-\sin^2 x}{\sin x}\right)\sec x.$$

Rewrite $j(x)$ as a single logarithm base 10 without negative exponents in any part of the expression. Your result should be of the form $\log_{10}(\text{expression})$.

(b)(ii) Rewrite $k(x)$ as a single term involving $\tan x$.

Question 4

2024 ▪ Q4

Directions:

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Question 4

(c) The function m is given by

$$m(x) = \cos^{-1}(\tan(2x)).$$

Find all values in the domain of m that yield an output value of 0.

Solution 4

(a)(i)

Mark scheme

- $g(x) = 10$:
  $e^{(x+3)} = 10 \Rightarrow \ln e^{(x+3)} = \ln 10 \Rightarrow x + 3 = \ln 10 \Rightarrow x = -3 + \ln 10$. 1 point for the correct solution with required supporting work (the step $x + 3 = \ln 10$ must appear, or the alternate change-of-base form $x = \frac{\log_b 10}{\log_b e} - 3$).

Solution 4

(a)(ii)

Mark scheme

- $h(x) = \pi/4$: $\arcsin(x/2) = \pi/4 \Rightarrow x/2 = \sin(\pi/4) = \frac{\sqrt{2}}{2}$ (or $\frac{1}{\sqrt{2}}$) $\Rightarrow x = \sqrt{2}$.
  1 point; supporting work must show $x/2 = \sin(\pi/4)$ or an equivalent evaluated form ($x/2 = \frac{\sqrt{2}}{2}$ or $x/2 = \frac{1}{\sqrt{2}}$).

Solution 4

(b)(i)

Mark scheme

- $j(x) = \log_{10}(8x^5) + \log_{10}(2x^2) - 9\log_{10} x = \log_{10}(8x^5 \cdot 2x^2) - \log_{10} x^9 = \log_{10}\left(\frac{16x^7}{x^9}\right) = \log_{10}\left(\frac{16}{x^2}\right)$, for $x > 0$. 1 point for the correct single-logarithm expression (domain restriction not required to be included and not scored); use of 'log' instead of '$\log_{10}$' is acceptable.

Solution 4

(b)(ii) Mark scheme

- $k(x) = \left(\frac{1-\sin^2 x}{\sin x}\right)\sec x = \left(\frac{\cos^2 x}{\sin x}\right)\left(\frac{1}{\cos x}\right) = \frac{\cos x}{\sin x} = \frac{1}{\tan x}$ (i.e. $\cot x$), with $\sin x \neq 0$, $\cos x \neq 0$. 1 point for the correct single term in $\tan x$ (domain restrictions not required).

Solution 4

(c) Mark scheme

- $m(x) = 0$:
  $\cos^{-1}(\tan(2x)) = 0 \Rightarrow \tan(2x) = \cos(0) = 1 \Rightarrow 2x = \frac{\pi}{4} + \pi n \Rightarrow x = \frac{\pi}{8} + \frac{\pi}{2}n$, where n is any integer (any letter other than x may be used for the integer parameter). 2 points: 1 point for one correct value of x (e.g. $x = \pi/8 + \pi n$), 1 point for all correct values of x (equivalently listing both $x = \pi/8 + \pi n$ and $x = 5\pi/8 + \pi n$) with no incorrect values included; 'where n is any integer' need not be stated explicitly.