

Past-paper walk-through

Sinusoidal Function Transformations ■ 3.6

eXercise

Questions in this set

- 2026 Q3
- 2025 Q3
- 2024 Q3

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- **//** separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = “or words to that effect” – the wording need not match.

Question 3

2026 ■ Q3

The figure shows a circular waterwheel that rotates in a counterclockwise direction at a constant rate and completes 1 revolution in 10 seconds. At time $t = 0$, point W on the edge of the waterwheel is at a height of 6 feet directly above the center of the waterwheel. The height of point W from the horizontal line through the center of the waterwheel periodically decreases and increases as the waterwheel moves.

[Diagram of a circular waterwheel partially submerged in water. The wheel has spokes radiating from a labeled "Center" point, and hatched paddles around the rim. Point W is labeled at the top of the wheel. An arrow labeled "Counterclockwise" indicates the direction of rotation. A vertical double-headed arrow labeled "Height of W " shows the vertical distance from the horizontal dashed line through the center up to W . The bottom portion of the wheel is submerged in wavy water. Note: Figure not drawn to scale.]

Question 3

The sinusoidal function h models the height of point W from the horizontal line through the center of the waterwheel, in feet, as a function of time t , in seconds. A positive value of $h(t)$ indicates W is above the line through the center; a negative value of $h(t)$ indicates W is below the line through the center.

(a) The graph of h and its dashed midline for two full cycles is shown. Five points, F , G , J , K , and P , are labeled on the graph. No scale is indicated, and no axes are presented.

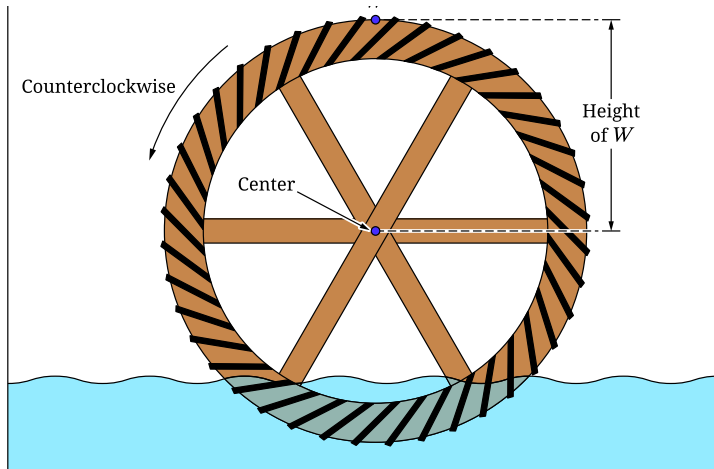
[Graph of h : a sinusoidal (cosine-shaped) curve with a dashed horizontal midline, showing two full cycles. No axes or scale are shown — only the curve and the midline. Point F is labeled at the first (leftmost) maximum of the curve. Point G is labeled where the curve crosses the midline, descending, between the first maximum and the first minimum. Point J is labeled at the minimum of the curve (between the two maxima). Point K is labeled where the curve crosses the midline, ascending, between

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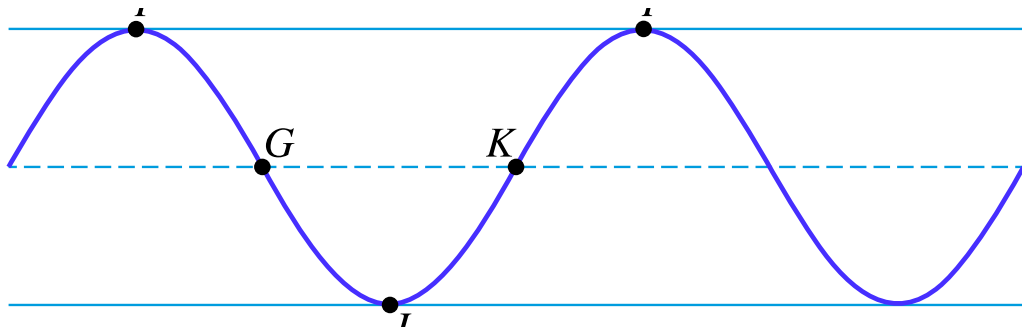
the minimum and the second maximum. Point P is labeled at the second maximum of the curve.]

Determine possible coordinates $(t, h(t))$ for the five points: F , G , J , K , and P .

Question 3



Question 3



Question 3

2026 ■ Q3

The figure shows a circular waterwheel that rotates in a counterclockwise direction at a constant rate and completes 1 revolution in 10 seconds. At time $t = 0$, point W on the edge of the waterwheel is at a height of 6 feet directly above the center of the waterwheel. The height of point W from the horizontal line through the center of the waterwheel periodically decreases and increases as the waterwheel moves.

[Diagram of a circular waterwheel partially submerged in water. The wheel has spokes radiating from a labeled "Center" point, and hatched paddles around the rim. Point W is labeled at the top of the wheel. An arrow labeled "Counterclockwise" indicates the direction of rotation. A vertical double-headed arrow labeled "Height of W " shows the vertical distance from the horizontal dashed line through the center up to W . The bottom portion of the wheel is submerged in wavy water. Note: Figure not drawn to scale.]

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Question 3

indicates W is above the line through the center; a negative value of $h(t)$ indicates W is below the line through the center.

(b) The function h can be written in the form $h(t) = a \sin(b(t + c)) + d$. Find values of constants a , b , c , and d .

Question 3

2026 ■ Q3

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indicates W is above the line through the center; a negative value of $h(t)$ indicates W is below the line through the center.

(c)(i) Refer to the graph of h in part A. The t -coordinate of J is t_1 , and the t -coordinate of K is t_2 .

On the interval (t_1, t_2) , which of the following is true about h ?

a. h is positive and increasing. b. h is positive and decreasing. c. h is negative and increasing. d. h is negative and decreasing.

(c)(ii) On the interval (t_1, t_2) , describe the concavity of the graph of h and determine whether the rate of change of h is increasing or decreasing.

Question 3

2025 ■ Q3

For a guitar to make a sound, the strings need to vibrate, or move up and down or back and forth, in a motion that can be modeled by a periodic function.

At time $t = 0$ seconds, point X on one vibrating guitar string starts at its highest position, 2 millimeters above its resting position. Then it passes through its resting position and moves to its lowest position, 2 millimeters below the resting position.

Point X then passes through its resting position and returns to 2 millimeters above the resting position. This motion occurs 200 times in 1 second.

The sinusoidal function h models how far point X is from its resting position, in millimeters, as a function of time t , in seconds. A positive value of $h(t)$ indicates the point is above the resting position; a negative value of $h(t)$ indicates the point is below the resting position.

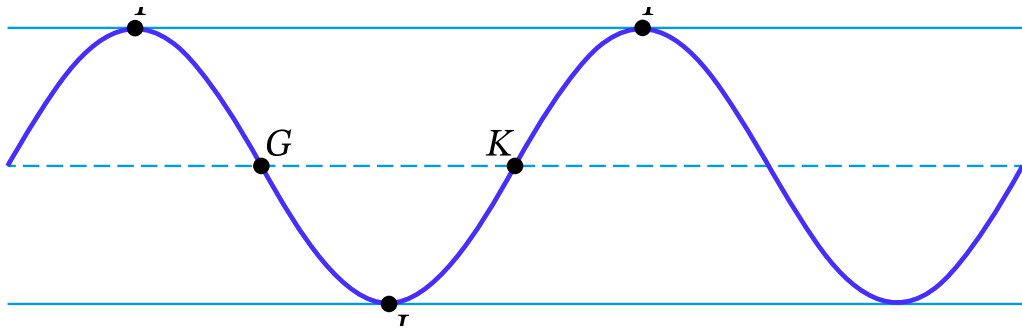
Question 3

(a) The graph of h and its dashed midline for two full cycles is shown. Five points, F , G , J , K , and P , are labeled on the graph. No scale is indicated, and no axes are presented.

Determine possible coordinates $(t, h(t))$ for the five points: F , G , J , K , and P .

[Graph of the sinusoidal function h over two full cycles, with a dashed horizontal midline and two solid horizontal lines marking the maximum and minimum levels; no numerical axes or scale shown. The curve starts at a maximum, labeled point F , descends crossing the midline at point G , continues down to a minimum labeled point J , rises back up crossing the midline at point K , and reaches the next maximum labeled point P , then the curve continues descending again through the end of the second cycle. Points in left-to-right order: F (first maximum), G (midline, descending), J (minimum), K (midline, ascending), P (second maximum).]

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(b) The function h can be written in the form $h(t) = a \sin(b(t + c)) + d$. Find values of constants a , b , c , and d .

Question 3

2025 ■ Q3

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(c)(i) Refer to the graph of h in part A. The t -coordinate of G is t_1 , and the t -coordinate of J is t_2 .

Question 3

On the interval (t_1, t_2) , which of the following is true about h ?

a. h is positive and increasing. b. h is positive and decreasing. c. h is negative and increasing. d. h is negative and decreasing.

(c)(ii) On the interval (t_1, t_2) , describe the concavity of the graph of h and determine whether the rate of change of h is increasing or decreasing.

Solution 3

(a) Mark scheme

- One full cycle: the period is $\frac{1}{200}$ s (the motion occurs 200 times per second), so a quarter period is $\frac{1}{800}$ s. Taking F at the starting maximum: $F(0, 2)$, $G(\frac{1}{800}, 0)$ (descending through rest), $J(\frac{2}{800}, -2)$ (minimum), $K(\frac{3}{800}, 0)$ (ascending through rest), $P(\frac{4}{800}, 2)$ (next maximum). t -coordinates may vary (e.g. shifted by a multiple of $\frac{4}{800}$) as long as they are evenly spaced by $\frac{1}{800}$ and follow this same pattern; correct $h(t)$ -coordinates (2, 0, -2, 0, 2 for F, G, J, K, P) are worth 1 point, and correct matching t -coordinates are worth 1 point. No supporting work is required.

Solution 3

(b) Mark scheme

- $h(t) = a \sin(b(t + c)) + d$. Since the period is $\frac{1}{200}$ s, $\frac{2\pi}{b} = \frac{1}{200} \Rightarrow b = 400\pi$. The midline is at 0, so $d = 0$. Using $a = 2$: $c = \frac{1}{800}$ (or $c = \frac{1}{800} + \frac{4}{800}k$ for any integer k), giving $h(t) = 2 \sin\left(400\pi\left(t + \frac{1}{800}\right)\right)$. (Equivalently, using $a = -2$: $c = -\frac{1}{800}$, giving $h(t) = -2 \sin\left(400\pi\left(t - \frac{1}{800}\right)\right)$.) Values of a and d are worth 1 point; values of b and c are worth 1 point. No supporting work is required.

Solution 3

(c)(i)

Mark scheme

- Choice (d): h is negative and decreasing on (t_1, t_2) (the interval between G , where $h = 0$ descending, and J , the minimum).

Solution 3

(c)(ii)

Mark scheme

- The graph of h is concave up on the interval (t_1, t_2) , and the rate of change of h is increasing on (t_1, t_2) . Both descriptions (concavity AND behavior of the rate of change) are required for the point; combining "concave up" with "function h is decreasing at an increasing rate" is also acceptable, but claims about the rate of change itself increasing/decreasing at an increasing/decreasing rate are not (that requires calculus).

Question 3

2024 ■ Q3

[Figure (not drawn to scale): a person pushes a car tire forward along level ground. A point labeled W is marked on the edge of the tire near the bottom.]

The tire of a car has a radius of 9 inches, and a person rolls the tire forward at a constant rate on level ground, as shown in the figure. Point W on the edge of the tire touches the ground at time $t = \frac{1}{2}$ second. The tire completes a full rotation, and the

next time W touches the ground is at time $t = \frac{5}{2}$ seconds. The maximum height of W above the ground is 18 inches. As the tire rolls, the height of W above the ground periodically increases and decreases.

The sinusoidal function h models the height of point W above the ground, in inches, as a function of time t , in seconds.

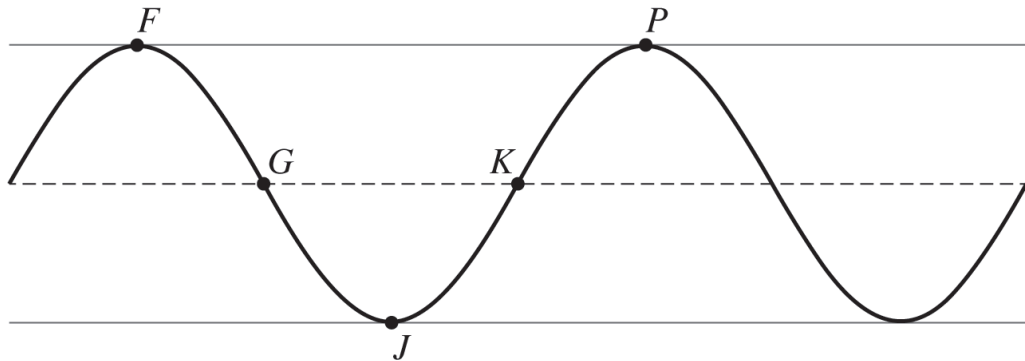
Question 3

(a) The graph of h and its dashed midline for two full cycles is shown. Five points, F , G , J , K , and P , are labeled on the graph. No scale is indicated, and no axes are presented.

[Graph: a sinusoidal curve over two full cycles with a dashed horizontal midline. Point F is at the first maximum (top of the first hump). Point G lies on the midline, on the descending part of the curve just after F . Point J is at the first minimum (bottom of the curve, between the two humps). Point K lies on the midline, on the ascending part of the curve just before the second maximum. Point P is at the second maximum (top of the second hump).]

Determine possible coordinates $(t, h(t))$ for the five points: F , G , J , K , and P .

Question 3



Question 3



Note: Figure not drawn to scale.

Question 3

2024 ■ Q3

[Figure (not drawn to scale): a person pushes a car tire forward along level ground. A point labeled W is marked on the edge of the tire near the bottom.]

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is at time $t = \frac{5}{2}$ seconds. The maximum height of W above the ground is 18 inches. As the tire rolls, the height of W above the ground periodically increases and decreases.

The sinusoidal function h models the height of point W above the ground, in inches, as a function of time t , in seconds.

(b) The function h can be written in the form $h(t) = a \sin(b(t + c)) + d$. Find values of constants a , b , c , and d .

Question 3

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is at time $t = \frac{5}{2}$ seconds. The maximum height of W above the ground is 18 inches. As the tire rolls, the height of W above the ground periodically increases and decreases.

The sinusoidal function h models the height of point W above the ground, in inches, as a function of time t , in seconds.

(c)(i) Refer to the graph of h in part (A). The t -coordinate of K is t_1 , and the t -coordinate of P is t_2 .

Question 3

On the interval (t_1, t_2) , which of the following is true about h ?

- a. h is positive and increasing. b. h is positive and decreasing. c. h is negative and increasing. d. h is negative and decreasing.

(c)(ii) Describe how the rate of change of h is changing on the interval (t_1, t_2) .

Solution 3

(a) Mark scheme

- One valid set of coordinates $(t, h(t))$ for one full cycle: $F = (3/2, 18)$, $G = (2, 9)$, $J = (5/2, 0)$, $K = (3, 9)$, $P = (7/2, 18)$ (an equally valid alternate cycle: $F = (-1/2, 18)$, $G = (0, 9)$, $J = (1/2, 0)$, $K = (1, 9)$, $P = (3/2, 18)$). No supporting work is required; t -coordinates may shift by any common integer multiple of the period (each must be of the form $-1/2 + 2k$, $0 + 2k$, $1/2 + 2k$, $1 + 2k$, or $3/2 + 2k$ for the same integer k). 2 points total: 1 point for correct $h(t)$ -coordinates (F and P at the max height 18; G and K at the midline height 9, less than F/P; J at the min height 0), 1 point for correct, consistently-spaced t -coordinates.

Solution 3

(b)

Mark scheme

- $h(t) = a \sin(b(t + c)) + d$ with $a = 9$, $b = \frac{2\pi}{2} = \pi$, $c = -1$, $d = 9$, giving $h(t) = 9 \sin(\pi(t - 1)) + 9$ (equivalently $a = -9$, $b = \pi$, $c = 0$, $d = 9$: $h(t) = -9 \sin(\pi t) + 9$; other forms are correct based on horizontal shifts/reflections, e.g. using cosine). No supporting work required. 2 points: 1 point for correct vertical-transformation values (a and d), 1 point for correct horizontal-transformation values (b and c).

Solution 3

(c)(i)

Mark scheme

- Choice (a): h is positive and increasing on the interval (t_1, t_2) (the interval from K , on the midline rising toward the next max, to P , the next maximum). 1 point for the correct letter choice or the equivalent written description 'positive and increasing' (if both are given, the written description is scored).

Solution 3

(c)(ii)

Mark scheme

- Because the graph of h is concave down on the interval (t_1, t_2) , the rate of change of h is decreasing on that interval (equivalently: h is increasing at a decreasing rate). 1 point; stating only 'the graph of h is concave down' without connecting it to the rate of change is NOT sufficient, and the subject 'the rate of change of h ' must be stated explicitly. This point cannot be earned if part (i) has any error.