

Exercise walk-through

Rates of Change in Polar Functions ■ 3.15

eXercise

You must be able to

- compute the **average rate of change** 平均变化率 of r with respect to θ
- interpret whether the curve spirals in or out

Method — Rate of change of r

The average rate of change of r over $[\theta_1, \theta_2]$ is

$$\frac{r(\theta_2) - r(\theta_1)}{\theta_2 - \theta_1}.$$

If r **increases** as θ increases, the curve spirals **outward**; if r decreases, it spirals inward.

Method — Example

$$r(\theta) = 3\theta: r(0) = 0, r(2) = 6; \text{ average rate} = \frac{6-0}{2-0} = 3.$$

Practice 2.1 ■ 1 mark

State the average rate of change of r over $[\theta_1, \theta_2]$.

Solution 2.1

Method: Rate of change of r

Worked steps

$$\frac{r(\theta_2) - r(\theta_1)}{\theta_2 - \theta_1} \quad \text{B1}.$$

Practice 2.2 ■ 1 mark

State what it means if r increases as θ increases.

Solution 2.2

Method: Rate of change of r

Worked steps

the curve spirals outward (moves farther from the origin) **B1**.

Practice 2.3 ■ 2 marks

For $r(\theta) = 2\theta$, find $r(0)$ and $r(\pi)$.

Solution 2.3

Worked steps

$$r(0) = 0; r(\pi) = 2\pi \quad \text{B1} \quad \text{B1}.$$

Exam-style 3.1 ■ 1 mark

The average rate of change of r on $[a, b]$ is

A $r(b) \cdot r(a)$

B $\frac{r(b) - r(a)}{b - a}$

C $r(b) + r(a)$

D $b - a$

Solution 3.1

Method: Rate of change of r

A $r(b) \cdot r(a)$

B $\frac{r(b) - r(a)}{b - a}$



C $r(b) + r(a)$

D $b - a$

Worked steps

B.

Exam-style 3.2 ■ 1 mark

If r increases as θ increases, the curve

A spirals inward

B spirals outward

C stays a circle

D is a straight line

Solution 3.2

Method: Rate of change of r

A spirals inward

B spirals outward



C stays a circle

D is a straight line

Worked steps

B.

Exam-style 3.3 ■ 1 mark

$$r(\theta) = 3\theta.$$

- (a) Find $r(0)$.
- (b) Find $r(2)$.
- (c) Find the average rate of change on $[0, 2]$.

Solution 3.3

Method: Rate of change of \$r\$

Worked steps

(a) 0 **B1**. (b) 6 **B1**. (c) $\frac{6-0}{2-0} = 3$ **B1**.