Exercise walk-through

Equivalent Representations of Trigonometric Functions ▪ 3.12

eXercise

You must be able to

- apply the **Pythagorean identity 勾股恒等式** $\sin^2 x + \cos^2 x = 1$
- use the even/odd symmetry of sine and cosine

Method — Key identities

- **Pythagorean**: $\sin^2 x + \cos^2 x = 1$, so $\cos^2 x = 1 - \sin^2 x$;
- **even/odd**: $\cos(-x) = \cos x$ (even), $\sin(-x) = -\sin x$ (odd).

Method — Example

If $\sin x = \frac{3}{5}$, then $\cos^2 x = 1 - \frac{9}{25} = \frac{16}{25}$, so $\cos x = \pm\frac{4}{5}$.

Practice 2.1 ▪ 1 mark

State the Pythagorean identity.

Solution 2.1

Worked steps

$\sin^2 x + \cos^2 x = 1$ **B1**.

Practice 2.2 ▪ 1 mark

Rewrite $1 - \cos^2 x$.

Solution 2.2

Worked steps

$\sin^2 x$ $\boxed{\textbf{B1}}$.

Practice 2.3 ▪ 2 marks

State $\sin(-x)$ in terms of $\sin x$.

Solution 2.3

Worked steps

$\sin(-x) = -\sin x$ **B1** **B1**.

Exam-style 3.1 ▪ 1 mark

The Pythagorean identity is

A $\sin x + \cos x = 1$

B $\sin^2 x + \cos^2 x = 1$

C $\sin^2 x - \cos^2 x = 1$

D $\sin x \cdot \cos x = 1$

Solution 3.1

A $\sin x + \cos x = 1$

B $\sin^2 x + \cos^2 x = 1$ ✓

C $\sin^2 x - \cos^2 x = 1$

D $\sin x \cdot \cos x = 1$

Worked steps

B.

Exam-style 3.2 ▪ 1 mark

$\cos(-x)$ equals

A $-\cos x$

B $\cos x$

C $\sin x$

D $-\sin x$

Solution 3.2

A $-\cos x$

B $\cos x$ ✓

C $\sin x$

D $-\sin x$

Worked steps

B.

Exam-style 3.3 ▪ 1 mark

Given $\sin x = \frac{3}{5}$.
(a) State $\sin^2 x$.
(b) Use the identity to find $\cos^2 x$.
(c) State a possible value of $\cos x$.

Solution 3.3

Worked steps

(a) $\frac{9}{25}$ **B1**. (b) $1 - \frac{9}{25} = \frac{16}{25}$ **B1**. (c) $\frac{4}{5}$ (or $-\frac{4}{5}$) **B1**.