Exercise walk-through

Trigonometric Equations and Inequalities ▪ 3.10

eXercise

You must be able to

- solve a simple trigonometric equation on a given interval
- explain why such equations have infinitely many solutions

Method — Solving trig equations

Solve $\sin x = c$ by finding one solution, then using symmetry and **periodicity 周期性** to get the rest $(+2\pi n)$.

Method — Example

$\sin x = \frac{1}{2}$ gives $x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$ on $[0, 2\pi)$, plus $2\pi n$ for all solutions.

Practice 2.1 ▪ 1 mark

Solve $\sin x = 0$ on $[0, 2\pi)$.

Solution 2.1

Worked steps

$x = 0$ and $x = \pi$ **B1**.

Practice 2.2 ▪ 1 mark

State why a trig equation has infinitely many solutions.

Solution 2.2

Worked steps

sine and cosine are periodic, so each solution repeats every 2π **B1**.

Practice 2.3 ▪ 2 marks

Solve $\cos x = 1$ on $[0, 2\pi)$.

Solution 2.3

Worked steps

$\cos x = 1$ at $x = 0$ only on $[0, 2\pi)$ **M1** **A1**.

Exam-style 3.1 ▪ 1 mark

The equation $\sin x = \frac{1}{2}$ has

A one solution in total

B two solutions in $[0, 2\pi)$

C no solution

D no solution in $[0, 2\pi)$

Solution 3.1

- **A** one solution in total
- **B** two solutions in $[0, 2\pi)$ ✓
- **C** no solution
- **D** no solution in $[0, 2\pi)$

Worked steps

B.

Exam-style 3.2 ▪ 1 mark

Trig equations have infinitely many solutions because sine and cosine are

- **A** linear
- **B** periodic
- **C** polynomial
- **D** rational

Solution 3.2

Method: Solving trig equations

A linear

B periodic ✓

C polynomial

D rational

Worked steps

B.

Exam-style 3.3 ▪ 1 mark

Solve $\sin x = 1$.

(a) State the solution on $[0, 2\pi)$.

(b) State the general solution (all x).

(c) State the amount added each time.

Solution 3.3

Method: Solving trig equations

Worked steps

(a) $x = \frac{\pi}{2}$ **B1**. (b) $x = \frac{\pi}{2} + 2\pi n$ **B1**. (c) 2π **B1**.