

Exercise walk-through

Inverse Functions ■ 2.8

eXercise

You must be able to

- find an **inverse function** 反函数 by swapping x and y and solving
- use that the graph of f^{-1} reflects f across the line $y = x$

Method — Inverses

f^{-1} **undoes** f : $(f \circ f^{-1})(x) = x$. Find it by swapping x and y and solving for y . The graph reflects across $y = x$. Only **one-to-one** functions have an inverse function.

Method — Example

$$f(x) = 2x + 1: \text{ swap } \rightarrow x = 2y + 1 \rightarrow y = \frac{x-1}{2}, \text{ so } f^{-1}(x) = \frac{x-1}{2}.$$

Practice 2.1 ■ 1 mark

State what an inverse function does.

Solution 2.1

Method: Inverses

Worked steps

it undoes f , returning the original input **B1**.

Practice 2.2 ■ 1 mark

State the line of reflection between f and f^{-1} .

Solution 2.2

Worked steps

the line $y = x$ **B1**.

Practice 2.3 ■ 2 marks

Find the inverse of $f(x) = x + 5$.

Solution 2.3

Worked steps

swap: $x = y + 5$; solve: $y = x - 5$, so $f^{-1}(x) = x - 5$ **M1** **A1**.

Exam-style 3.1 ■ 1 mark

The graph of f^{-1} is the reflection of f across

A the x -axis

B the y -axis

C the line $y = x$

D the origin

Solution 3.1

Method: Inverses

- A the x -axis
- B the y -axis
- C the line $y = x$
- D the origin



Worked steps

C.

Exam-style 3.2 ■ 1 mark

A function has an inverse function only if it is

A increasing

B one-to-one

C linear

D positive

Solution 3.2

Method: Inverses

A increasing

B one-to-one



C linear

D positive

Worked steps

B.

Exam-style 3.3 ■ 1 mark

$$f(x) = 3x - 6.$$

- (a) Swap x and y .
- (b) Solve for y .
- (c) State $f^{-1}(x)$.

Solution 3.3

Worked steps

$$(a) \ x = 3y - 6 \quad \text{B1}. \quad (b) \ y = \frac{x+6}{3} \quad \text{B1}. \quad (c) \ f^{-1}(x) = \frac{x+6}{3} \quad \text{B1}.$$