Past-paper walk-through

Logarithmic Function Context and Data Modeling ▪ 2.14

eXercise

Questions in this set

- 2024 Q2

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 2

2024 ▪ Q2

On the initial day of sales ($t = 0$) for a new video game, there were 40 thousand units of the game sold that day. Ninety-one days later ($t = 91$), there were 76 thousand units of the game sold that day.

The number of units of the video game sold on a given day can be modeled by the function G given by $G(t) = a + b\ln(t+1)$, where $G(t)$ is the number of units sold, in thousands, on day t since the initial day of sales.

(a)(i) Use the given data to write two equations that can be used to find the values for constants a and b in the expression for $G(t)$.

(a)(ii) Find the values for a and b as decimal approximations.

Question 2

2024 ▪ Q2

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(b)(i) Use the given data to find the average rate of change of the number of units of the video game sold, in thousands per day, from $t = 0$ to $t = 91$ days. Express your answer as a decimal approximation. Show the computations that lead to your answer.

(b)(ii) Use the average rate of change found in (i) to estimate the number of units of the video game sold, in thousands, on day $t = 50$. Show the work that leads to your answer.

Question 2

(b)(iii) Let A_t represent the estimate of the number of units of the video game sold, in thousands, using the average rate of change found in (i). For A_{50}, found in (ii), it can be shown that $A_{50} < G(50)$. Explain why, in general, $A_t < G(t)$ for all t, where $0 < t < 91$.

Question 2

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(c) The makers of the video game reported that daily sales of the video game decreased each day after $t = 91$. Explain why the error in the model G increases after $t = 91$.

Solution 2

(a)(i)

Mark scheme

- Because $G(0) = 40$ and $G(91) = 76$, two equations to find a and b are $a + b\ln(0+1) = 40$ and $a + b\ln(91+1) = 76$. 1 point for presenting two equations involving a and b that use the given input-output pairs.

Solution 2

(a)(ii)

Mark scheme

- From $a + b \ln 1 = 40$: $a = 40$. Then $b = \dfrac{76 - 40}{\ln 92} = 7.961451$. So $G(t) = 40 + 7.961 \ln(t + 1)$. 1 point for correct decimal values of a and b (with or without supporting work; a translation to actual units, $a = 40{,}000$ and $b = 7961$, is also acceptable).

Solution 2

(b)(i)

Mark scheme

- Average rate of change $= \dfrac{G(91) - G(0)}{91 - 0} = \dfrac{76 - 40}{91} = 0.395604 \approx 0.396$ (or 0.395) thousand units per day. 1 point for the correct decimal approximation shown via a quotient using the given data values.

Solution 2

(b)(ii)

Mark scheme

- Using the average rate of change $r = 0.395604$ and the secant line through $(0, G(0))$ and $(91, G(91))$: $y = G(0) + r(t - 0)$. For $t = 50$: $y = 40 + 0.395604(50) = 59.780$ (equivalently $y = 76 + r(50 - 91) = 59.780$). The estimated number of units sold on day $t = 50$ is approximately 59.780 thousand (59 or 60 thousand also accepted). 1 point for a correct estimate consistent with (i), with supporting work.

Solution 2

(b)(iii)

Mark scheme

- A_t is the y-coordinate of a point on the secant line through $(0, G(0))$ and $(91, G(91))$. Because the graph of G is concave down on $(0, 91)$, this secant line lies below the graph of G on that interval. Therefore the estimate A_t is less than $G(t)$ for all t on $(0, 91)$. 1 point requires reasoning that references BOTH that G is concave down (or its rate of change is decreasing) AND the use of a secant line on $0 < t < 91$ (or a linear function referencing endpoints 0 and 91).

Solution 2

(c)

Mark scheme

- On day $t = 91$, the actual daily sales and $G(91)$ are equal. For $t > 91$, actual daily sales are decreasing while G is increasing. Therefore the absolute value of the difference between actual daily sales and the daily sales predicted by G increases each day for $t > 91$, so the error in the model increases. 1 point requires an implicit or explicit connection between 'the function model G is increasing' and 'daily sales are (actually) decreasing.'