Past-paper walk-through

Exponential and Logarithmic Equations and Inequalities ▪ 2.13

eXercise

Questions in this set

- 2026 Q1
- 2026 Q4
- 2025 Q4
- 2024 Q1

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 1

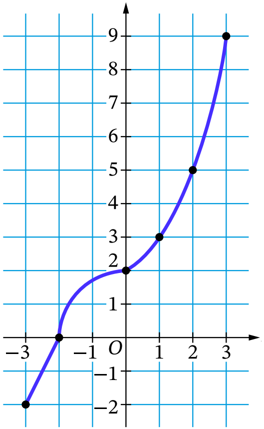
2026 ▪ Q1

The figure shows the graph of the increasing function f on its domain of $-3 \leq x \leq 3$. [Graph of f: a smooth increasing S-shaped curve on a coordinate grid, x-axis from -3 to 3, y-axis from -2 to 9 (gridlines at every integer). Labeled points on the curve: $(-3, -2)$, $(-1, 1)$, $(0, 1.8)$, $(1, 3)$, $(2, 5)$, $(3, 9)$. The curve rises slowly near the middle (around $x = 0$) and rises steeply toward both ends.]
The function g is given by $g(x) = -4.792 + \ln(6x - 6)$.
(a)(i) The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(2)$ as a decimal approximation, or indicate that it is not defined. Show the work that leads to your answer.
(a)(ii) Find all values of x for which $f(x) = 3$, or indicate that there are no such values.
(b)(i) Find all values of x, as decimal approximations, for which $g(x) = -1.5$, or indicate that there are no such values.

Question 1

(b)(ii) Determine the behavior of g as x-values decrease and get arbitrarily close to 1. Express your answer using the mathematical notation of a limit.
(c)(i) Is the function f invertible?
(c)(ii) Give a reason for your answer in part C (i) based on properties of the function f. Refer to points on the graph of f in your reasoning.

Question 1



Graph of f

Question 4

2026 ▪ Q4

Directions:
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians. - Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator. - Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms. - For each part of the question, show the work that leads to your answers.

(a)(i) The functions g and h are given by

Question 4

$$g(x) = e^{2x}$$

$$h(x) = \log_2(5x)$$

Solve $g(x) = \dfrac{1}{e^6}$ for values of x in the domain of g.

(a)(ii) Solve $h(x) = 3$ for values of x in the domain of h.

Question 4

2026 ▪ Q4

Directions:

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians. - Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator. - Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms. - For each part of the question, show the work that leads to your answers.

(b)(i) The functions j and k are given by

Question 4

$$j(x) = 7^{(3x+1)} \cdot 7^x$$

$$k(x) = \sin(2x) \sec x$$

Rewrite $j(x)$ as an expression of the form $7^{(\text{expression})}$.

(b)(ii) Rewrite $k(x)$ as an expression in which $\sin x$ appears exactly once and no other trigonometric functions are involved.

Question 4

2026 ▪ Q4

Directions:

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians. - Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator. - Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms. - For each part of the question, show the work that leads to your answers.

(c) The function m is given by $m(x) = \tan^2(3x)$.

Find all input values for m in the interval $\left[0, \frac{\pi}{2}\right]$ that yield an output value of 1.

Question 4

2025 ▪ Q4

Directions:
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians. - Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator. - Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms. - For each part of the question, show the work that leads to your answers.

Question 4

(a)(i) The functions g and h are given by

$$g(x) = 2\log_3 x$$

$$h(x) = 4\cos^2 x$$

Solve $g(x) = 4$ for values of x in the domain of g.

(a)(ii) Solve $h(x) = 3$ for values of x in the interval $\left[0, \dfrac{\pi}{2}\right]$.

Question 4

2025 ▪ Q4

Directions:

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians. - Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator. - Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms. - For each part of the question, show the work that leads to your answers.

Question 4

(b)(i) The functions j and k are given by

$$j(x) = \log_2 x + 3\log_2 2$$

$$k(x) = \frac{6}{\tan x\,(\csc^2 x - 1)}$$

Rewrite $j(x)$ as a single logarithm base 2 without negative exponents in any part of the expression. Your result should be of the form $\log_2$(expression).

(b)(ii) Rewrite $k(x)$ as an expression in which $\tan x$ appears exactly once and no other trigonometric functions are involved.

Question 4

2025 ▪ Q4

Directions:

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians. - Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator. - Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms. - For each part of the question, show the work that leads to your answers.

(c) The function m is given by $m(x) = e^{2x} - e^x - 12$. Find all input values in the domain of m that yield an output value of 0.

Solution 4

(a)(i)

Mark scheme

- $g(x) = 2\log_3 x = 4 \Rightarrow \log_3 x = 2 \Rightarrow x = 3^2 = 9.$

Solution 4

(a)(ii)

Mark scheme

- $h(x) = 4\cos^2 x = 3 \Rightarrow \cos^2 x = \frac{3}{4} \Rightarrow \cos x = \pm\frac{\sqrt{3}}{2}$. Since $x \in \left[0, \frac{\pi}{2}\right]$ (first quadrant, cosine positive), $x = \frac{\pi}{6}$.

Solution 4

(b)(i)

Mark scheme

- $j(x) = \log_2 x + 3\log_2 2 = \log_2 x + \log_2 2^3 = \log_2(8x),\ x > 0.$

Solution 4

(b)(ii)

Mark scheme

- $k(x) = \dfrac{6}{\tan x(\csc^2 x - 1)}$. Using the Pythagorean identity $\csc^2 x - 1 = \cot^2 x$:
  $k(x) = \dfrac{6}{\tan x \cot^2 x} = \dfrac{6}{\cot x} = 6\tan x$ (with $\tan x \neq 0$, $\cot x \neq 0$).

Solution 4

(c)

Mark scheme

- $m(x) = e^{2x} - e^x - 12 = 0$. Let $y = e^x$, giving $y^2 - y - 12 = 0$ (quadratic form with e^x, worth 1 point) $\Rightarrow (y-4)(y+3) = 0 \Rightarrow y = 4$ or $y = -3$. Since $y = e^x > 0$, discard $y = -3$; $e^x = 4 \Rightarrow x = \ln 4$ (value of x, worth 1 point, with no incorrect values included).

Question 1

2024 ▪ Q1

[Graph of f: x-axis from -4 to 4, y-axis from -2 to 4, gridlines at each integer. The curve begins at an open/filled point near $(-3, 3.5)$, drops steeply down through $(-2, 0)$ to a minimum near $(-1.7, -1.4)$, rises up crossing near $x = -1$ and $x = 0$ passing through $(0, 1)$, continues rising to a maximum near $(2, 3.5)$, then decreases sharply ending at a point near $(3.3, -1.6)$. Labeled "Graph of *f*".]

The figure shows the graph of the function *f* on its domain of $-3.5 \leq x \leq 3.5$. The points $(-3, 1)$, $(0, 1)$, and $(3, 1)$ are on the graph of *f*. The function *g* is given by $g(x) = 2.916 \cdot (0.7)^x$.

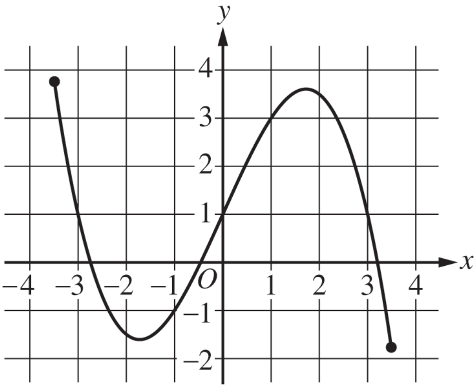
(a)(i) The function *h* is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(3)$ as a decimal approximation, or indicate that it is not defined.

(a)(ii) Find all values of *x* for which $f(x) = 1$, or indicate that there are no such values.

Question 1

(b)(i) Find all values of x, as decimal approximations, for which $g(x) = 2$, or indicate that there are no such values.
(b)(ii) Determine the end behavior of g as x increases without bound. Express your answer using the mathematical notation of a limit.
(c)(i) Determine if f has an inverse function.
(c)(ii) Give a reason for your answer based on the definition of a function and the graph of $y = f(x)$.

Question 1



Graph of f

Solution 1

(a)(i)

Mark scheme

- $h(3) = g(f(3)) = g(1) = 2.041$, using $g(x) = 2.916 \cdot (0.7)^x$ and $f(3) = 1$ read from the graph. 1 point for the correct decimal approximation 2.041 (decimals must be correct to three places; the first decimal-presentation error in the whole question is forgiven).

Solution 1

(a)(ii)

Mark scheme

- From the graph, $f(x) = 1$ when $x = -3$, $x = 0$, and $x = 3$ (the three marked points $(-3, 1), (0, 1), (3, 1)$). 1 point for all three correct values with no incorrect values included.

Solution 1

(b)(i)

Mark scheme

- Solve $g(x) = 2$: $2.916(0.7)^x = 2 \Rightarrow x = 1.057$ (decimal approximation). 1 point for the correct value with no incorrect values included.

Solution 1

(b)(ii)

Mark scheme

- As x increases without bound, the output values of g get arbitrarily close to 0, so $\lim_{x\to\infty} g(x) = 0$. 1 point requires a correct limit statement with all four components: 'lim', '$x \to \infty$', the function g, and the value 0 (e.g. $\lim_{x\to\infty} g(x) = 0$).

Solution 1

(c)(i)

Mark scheme

- f does not have an inverse function on its domain $-3.5 \leq x \leq 3.5$. 1 point for the correct answer.

Solution 1

(c)(ii)

Mark scheme

- There are output values of f that are not mapped from unique input values; for example $f(-3) = f(0) = f(3) = 1$. 1 point requires an implicit or explicit reference to the definition of a function AND support by referencing specific function values (e.g. citing $f(-3) = f(0) = 1$); saying only 'f fails the horizontal line test' or 'f is not one-to-one' is NOT sufficient. This point cannot be earned if part (i) has any error.