

Exercise walk-through

Rational Functions and Vertical Asymptotes ■ 1.9

eXercise

You must be able to

- identify a **vertical asymptote** 垂直渐近线 at $x = a$
- describe the behaviour of $r(x)$ near a vertical asymptote

Method — Vertical asymptotes

A rational function has a **vertical asymptote** at $x = a$ if a is a zero of the **denominator** but **not** of the numerator (or has higher multiplicity in the denominator).

Method — Behaviour near it

Near $x = a$, the denominator is close to 0, so the values **increase or decrease without bound**: $\lim_{x \rightarrow a^+} r(x) = \pm\infty$.

For $r(x) = \frac{1}{x-3}$, the vertical asymptote is $x = 3$.

Practice 2.1 ■ 1 mark

State the condition for a vertical asymptote at $x = a$.

Solution 2.1

Worked steps

a is a zero of the denominator but not of the numerator **B1**.

Practice 2.2 ■ 1 mark

For $r(x) = \frac{1}{x-3}$, state the vertical asymptote.

Solution 2.2

Method: Behaviour near it

Worked steps

$$x = 3 \text{ (B1).}$$

Practice 2.3 ■ 1 mark

State what happens to $r(x)$ near a vertical asymptote.

Solution 2.3

Method: Behaviour near it

Worked steps

the values increase or decrease without bound **B1**.

Exam-style 3.1 ■ 1 mark

A rational function has a vertical asymptote where the _____ is zero (but not the numerator).

A numerator

B denominator

C leading term

D degree

Solution 3.1

Method: Vertical asymptotes

- A numerator
- B denominator
- C leading term
- D degree



Worked steps

B.

Exam-style 3.2 ■ 1 mark

Near a vertical asymptote, the function values

- A** approach 0
- B** increase or decrease without bound
- C** stay constant
- D** equal the mean

Solution 3.2

Method: Behaviour near it

- A approach 0
- B increase or decrease without bound
- C stay constant
- D equal the mean



Worked steps

B.

Exam-style 3.3 ■ 1 mark

$$r(x) = \frac{x+2}{x-5}.$$

- (a) State the vertical asymptote.
- (b) State the zero.
- (c) State the domain restriction.

Solution 3.3

Worked steps

(a) $x = 5$ B1. (b) $x = -2$ B1. (c) $x \neq 5$ B1.