Exercise walk-through

Rational Functions and End Behavior ▪ 1.7

eXercise

You must be able to

- describe a **rational function 有理函数** as a quotient of polynomials
- find its end behavior from the ratio of the **leading terms**
- identify a **horizontal asymptote 水平渐近线** where one exists

Method — Rational functions

A **rational function** $r(x) = \dfrac{p(x)}{q(x)}$ is a quotient of two polynomials.

Method — End behavior

For large $|x|$ each polynomial is dominated by its leading term, so the end behavior is the ratio of the leading terms:

- degree(top) $<$ degree(bottom) $\to$ horizontal asymptote $y = 0$;
- degrees **equal** $\to$ $y = \dfrac{\text{leading coeff of top}}{\text{leading coeff of bottom}}$;
- degree(top) $>$ degree(bottom) $\to$ **no** horizontal asymptote.

Practice 2.1 ▪ 1 mark

State what determines the end behavior of a rational function.

Solution 2.1

Method: Rational functions

Worked steps

the ratio of the leading terms of the numerator and denominator B1.

Practice 2.2 ▪ 1 mark

For $r(x) = \dfrac{2x}{x^2 + 1}$, state the horizontal asymptote.

Solution 2.2

Method: End behavior

Worked steps

$y = 0$ (numerator degree is smaller) B1.

Practice 2.3 ▪ 2 marks

For $r(x) = \dfrac{3x^2}{x^2+1}$, state the horizontal asymptote.

Solution 2.3

Method: End behavior

Worked steps

$y = 3$ (equal degrees, ratio of leading coefficients $3/1$) **M1** **A1**.

Exam-style 3.1 ▪ 1 mark

The end behavior of a rational function is found from the ratio of the

A constant terms

B leading terms

C zeros

D holes

Solution 3.1

A constant terms

B leading terms ✓

C zeros

D holes

Worked steps

B.

Exam-style 3.2 ▪ 1 mark

If the numerator degree is less than the denominator degree, the horizontal asymptote is

A $y = 1$

B $y = 0$

C none

D $y = x$

Solution 3.2

Method: End behavior

A $y = 1$

B $y = 0$ ✓

C none

D $y = x$

Worked steps

B.

Exam-style 3.3 ▪ 2 marks

$r(x) = \dfrac{4x^2 + 1}{2x^2 - 3}$.

(a) State the horizontal asymptote.

(b) State whether the degrees are equal.

Solution 3.3

Method: End behavior

Worked steps

(a) $y = \dfrac{4}{2} = 2$ M1 A1. (b) yes (both degree 2) B1.