Past-paper walk-through

Polynomial Functions and End Behavior ▪ 1.6

eXercise

Questions in this set

- 2025 Q1

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 1

2025 ▪ Q1

The function f is decreasing and is defined for all real numbers. The table gives values for $f(x)$ at selected values of x.

x	-2	-1	0	1	2
$f(x)$	14	7	3.5	1.75	0.875

The function g is given by $g(x) = -0.167x^3 + x^2 - 1.834$.

Question 1

(a)(i) The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(1)$ as a decimal approximation, or indicate that it is not defined. Show the work that leads to your answer.
(a)(ii) Find the value of $f^{-1}(3.5)$, or indicate that it is not defined.
(b)(i) Find all values of x, as decimal approximations, for which $g(x) = 0$, or indicate that there are no such values.
(b)(ii) Determine the end behavior of g as x increases without bound. Express your answer using the mathematical notation of a limit.
(c)(i) Based on the table, which of the following function types best models function f: linear, quadratic, exponential, or logarithmic?

Question 1

(c)(ii) Give a reason for your answer in part C (i) based on the relationship between the change in the output values of f and the change in the input values of f. Refer to the values in the table in your reasoning.

Question 1

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A.

i. The function h is defined by $h(x) = (g \circ f)(x) = g\big(f(x)\big)$. Find the value of $h(1)$ decimal approximation, or indicate that it is not defined. Show the work that le

Solution 1

(a)(i)

Mark scheme

- $h(1) = g(f(1)) = g(1.75)$. From the table, $f(1) = 1.75$. Then $g(1.75) = -0.167(1.75)^3 + (1.75)^2 - 1.834 \approx 0.333$. So $h(1) = 0.333$ (decimal approximation, accurate to three places). The point is earned for the correct decimal approximation of 0.333 with supporting work showing $f(1)$ or $g(1.75)$ or 1.75.

(a)(ii)

Mark scheme

- From the table, $f(0) = 3.5$, so $f^{-1}(3.5) = 0$. No supporting work is required for this point; a response of "0" earns it.

Solution 1

(b)(i)

Mark scheme

- Solve $g(x) = 0$: $-0.167x^3 + x^2 - 1.834 = 0$. The three real solutions (decimal approximations) are $x = -1.233$, $x = 1.578$, $x = 5.643$. No supporting work is required; all three values must be given.

Solution 1

(b)(ii)

Mark scheme

- As x increases without bound, the output values of g decrease without bound (the leading term $-0.167x^3$ dominates and is negative). So $\lim_{x \to \infty} g(x) = -\infty$. The correct limit statement needs four components: "lim", "$x \to \infty$", the function g, and $-\infty$.

Solution 1

(c)(i)

Mark scheme

- An exponential function best models f.

Solution 1

(c)(ii) Mark scheme

- In the table, the input-value intervals all have length 1, and the ratios of successive output values are constant: $\frac{f(-1)}{f(-2)} = \frac{f(0)}{f(-1)} = \frac{f(1)}{f(0)} = \frac{f(2)}{f(1)} = 0.5$. Because the successive output values over equal-length input intervals are proportional (by a constant factor of 0.5), an exponential model is best. The reasoning must reference the table values and show the 0.5 ratio applies to more than one pair of successive outputs; referencing "exponential regression" or r/r^2 values alone is not sufficient.