Exercise walk-through

Polynomial Functions and End Behavior ▪ 1.6

eXercise

You must be able to

- describe the **end behavior 端点行为** of a polynomial from its leading term
- use **limit notation 极限记号** such as $\lim_{x\to\infty} p(x) = \infty$

Method — End behavior

The **end behavior** of a polynomial is set by its **leading term** $a_n x^n$:

- **even** n, $a_n > 0$: both ends up.
- **odd** n, $a_n > 0$: left end down, right end up.

Method — Limit notation

"Output increases without bound as x increases" is written $\lim_{x\to\infty} p(x) = \infty$. For $p(x) = x^3$: $\lim_{x\to\infty} p(x) = \infty$ and $\lim_{x\to-\infty} p(x) = -\infty$.

Practice 2.1 ▪ 1 mark

State what determines a polynomial's end behavior.

Solution 2.1

Method: End behavior

Worked steps

its leading term (degree and leading coefficient) B1.

Practice 2.2 ▪ 2 marks

For $p(x) = x^3$, state the end behavior as $x \to \infty$ and as $x \to -\infty$.

Solution 2.2

Worked steps

as $x \to \infty$, $p(x) \to \infty$; as $x \to -\infty$, $p(x) \to -\infty$ B1 B1.

Practice 2.3 ▪ 1 mark

Write the limit notation for “output increases without bound as x increases”.

Solution 2.3

Method: Limit notation

Worked steps

$\lim_{x\to\infty} p(x) = \infty$ B1.

Exam-style 3.1 ▪ 1 mark

The end behavior of a polynomial is determined by its

A constant term

B leading term

C middle term

D zero

Solution 3.1

Method: End behavior

A constant term

B leading term ✓

C middle term

D zero

Worked steps

B.

Exam-style 3.2 ▪ 1 mark

For $p(x) = -2x^4$, as $x \to \infty$ the output

A $\to \infty$

B $\to -\infty$

C $\to 0$

D oscillates

Solution 3.2

Method: Limit notation

A $\to \infty$

B $\to -\infty$ ✓

C $\to 0$

D oscillates

Worked steps

B — an even degree with a negative leading coefficient sends both ends to $-\infty$.

Exam-style 3.3 ▪ 1 mark

$p(x) = x^3$.
(a) State $\lim_{x \to \infty} p(x)$.
(b) State $\lim_{x \to -\infty} p(x)$.
(c) State whether both ends go the same way.

Solution 3.3

Method: End behavior

Worked steps

(a) ∞ B1. (b) $-\infty$ B1. (c) no (opposite directions) B1.