Exercise walk-through

Polynomial Functions and Complex Zeros ▪ 1.5

eXercise

You must be able to

- relate a **zero 零点** a to the linear factor $(x - a)$
- describe the **multiplicity 重数** of a zero
- use that a degree-n polynomial has exactly n complex zeros (with multiplicity)

Method — Zeros and factors

If $p(a) = 0$ then a is a **zero** of p, and (for real a) $(x - a)$ is a **factor**.

Method — Multiplicity

If a factor $(x - a)$ is repeated n times, the zero has **multiplicity** n. A polynomial of degree n has exactly n complex zeros, counting multiplicities.

Method — Example

$p(x) = (x-2)^2(x+1)$ has zeros $x = 2$ (multiplicity 2) and $x = -1$ (multiplicity 1); degree 3.

Practice 2.1 ▪ 1 mark

State the relationship between a zero a and the factor $(x - a)$.

Solution 2.1

Worked steps

$(x - a)$ is a factor of p if and only if a is a zero of p **B1**.

Practice 2.2 ▪ 1 mark

State how many complex zeros a degree-4 polynomial has (counting multiplicity).

Solution 2.2

Method: Multiplicity

Worked steps

4 **B1**.

Practice 2.3 ▪ 2 marks

For $p(x) = (x-2)^2(x+1)$, state the zeros and their multiplicities.

Solution 2.3

Method: Multiplicity

Worked steps

$x = 2$ (multiplicity 2); $x = -1$ (multiplicity 1) **B1** **B1**.

Exam-style 3.1 ▪ 1 mark

If $p(a) = 0$, then $(x - a)$ is a

A vertical asymptote

B factor of p

C hole

D leading term

Solution 3.1

A vertical asymptote

B factor of p ✓

C hole

D leading term

Worked steps

B.

Exam-style 3.2 ▪ 1 mark

A degree-5 polynomial has ________ complex zeros counting multiplicity.

A 1

B 3

C 5

D 10

Solution 3.2

Method: Multiplicity

A 1

B 3

C 5 ✓

D 10

Worked steps

C.

Exam-style 3.3 ▪ 1 mark

$p(x) = (x - 3)^3(x + 2)$.

(a) State the zeros.

(b) State the multiplicity of $x = 3$.

(c) State the degree.

Solution 3.3

Worked steps

(a) $x = 3$ and $x = -2$ **B1**. (b) 3 **B1**. (c) 4 **B1**.