

Exercise walk-through

Rates of Change ■ 1.2

eXercise

You must be able to

- calculate the **average rate of change** 平均变化率 over an interval
- describe the **rate of change at a point** 某点变化率

Method — Average rate of change

Over an interval $[a, b]$:

$$\text{AROC} = \frac{f(b) - f(a)}{b - a},$$

the ratio of the change in output to the change in input.

Method — Rate of change at a point

The rate of change **at a point** describes how fast the output would change there; it can be **approximated** by average rates over small intervals containing the point. For

$$f(1) = 3, f(4) = 12: \text{AROC} = \frac{12 - 3}{4 - 1} = 3.$$

Practice 2.1 ■ 1 mark

Write the formula for the average rate of change over $[a, b]$.

Solution 2.1

Worked steps

$$\frac{f(b) - f(a)}{b - a} \quad \text{B1}.$$

Practice 2.2 ■ 2 marks

If $f(2) = 5$ and $f(6) = 17$, find the average rate of change over $[2, 6]$.

Solution 2.2

Worked steps

$$\frac{17 - 5}{6 - 2} = \frac{12}{4} = 3 \quad \text{M1 A1}.$$

Practice 2.3 ■ 1 mark

State what the rate of change at a point describes.

Solution 2.3

Method: Rate of change at a point

Worked steps

how fast the output would change at that input value **B1**.

Exam-style 3.1 ■ 1 mark

The average rate of change of f over $[a, b]$ is

A $f(b) - f(a)$

B $\frac{f(b) - f(a)}{b - a}$

C $b - a$

D $f(b) + f(a)$

Solution 3.1

A $f(b) - f(a)$

B $\frac{f(b) - f(a)}{b - a}$



C $b - a$

D $f(b) + f(a)$

Worked steps

B.

Exam-style 3.2 ■ 1 mark

The rate of change at a point can be approximated using

- A** the range
- B** average rates over small intervals
- C** the domain
- D** the degree

Solution 3.2

Method: Rate of change at a point

- A the range
- B average rates over small intervals
- C the domain
- D the degree



Worked steps

B.

Exam-style 3.3 ■ 2 marks

$f(0) = 2$ and $f(5) = 22$, where f is distance (m) and x is time (s).

- (a) Find the average rate of change over $[0, 5]$.
- (b) State its units interpretation.

Solution 3.3

Worked steps

(a) $\frac{22 - 2}{5 - 0} = 4$ **M1** **A1**. (b) 4 metres per second (average speed) **B1**.