Past-paper walk-through

Change in Tandem ▪ 1.1

eXercise

Questions in this set

- 2024 Q1

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 1

2024 ▪ Q1

[Graph of f: x-axis from -4 to 4, y-axis from -2 to 4, gridlines at each integer. The curve begins at an open/filled point near $(-3, 3.5)$, drops steeply down through $(-2, 0)$ to a minimum near $(-1.7, -1.4)$, rises up crossing near $x = -1$ and $x = 0$ passing through $(0, 1)$, continues rising to a maximum near $(2, 3.5)$, then decreases sharply ending at a point near $(3.3, -1.6)$. Labeled "Graph of f".]

The figure shows the graph of the function f on its domain of $-3.5 \leq x \leq 3.5$. The points $(-3, 1)$, $(0, 1)$, and $(3, 1)$ are on the graph of f. The function g is given by $g(x) = 2.916 \cdot (0.7)^x$.

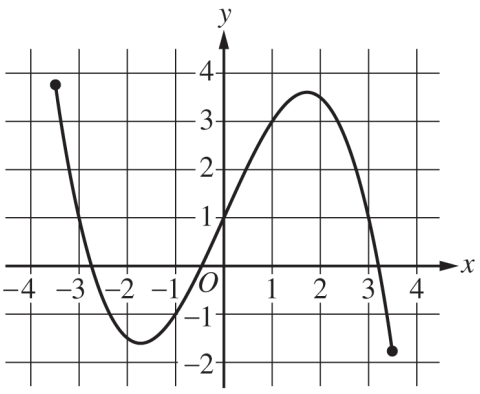
(a)(i) The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(3)$ as a decimal approximation, or indicate that it is not defined.

(a)(ii) Find all values of x for which $f(x) = 1$, or indicate that there are no such values.

Question 1

(b)(i) Find all values of x, as decimal approximations, for which $g(x) = 2$, or indicate that there are no such values.
(b)(ii) Determine the end behavior of g as x increases without bound. Express your answer using the mathematical notation of a limit.
(c)(i) Determine if f has an inverse function.
(c)(ii) Give a reason for your answer based on the definition of a function and the graph of $y = f(x)$.

Question 1



Graph of f

Solution 1

(a)(i)

Mark scheme

- $h(3) = g(f(3)) = g(1) = 2.041$, using $g(x) = 2.916 \cdot (0.7)^x$ and $f(3) = 1$ read from the graph. 1 point for the correct decimal approximation 2.041 (decimals must be correct to three places; the first decimal-presentation error in the whole question is forgiven).

Solution 1

(a)(ii)

Mark scheme

- From the graph, $f(x) = 1$ when $x = -3$, $x = 0$, and $x = 3$ (the three marked points $(-3, 1), (0, 1), (3, 1)$). 1 point for all three correct values with no incorrect values included.

Solution 1

(b)(i)

Mark scheme

- Solve $g(x) = 2$: $2.916(0.7)^x = 2 \Rightarrow x = 1.057$ (decimal approximation). 1 point for the correct value with no incorrect values included.

Solution 1

(b)(ii)

Mark scheme

- As x increases without bound, the output values of g get arbitrarily close to 0, so $\lim_{x\to\infty} g(x) = 0$. 1 point requires a correct limit statement with all four components: 'lim', '$x \to \infty$', the function g, and the value 0 (e.g. $\lim_{x\to\infty} g(x) = 0$).

Solution 1

(c)(i)

Mark scheme

- f does not have an inverse function on its domain $-3.5 \leq x \leq 3.5$. 1 point for the correct answer.

Solution 1

(c)(ii)

Mark scheme

- There are output values of f that are not mapped from unique input values; for example $f(-3) = f(0) = f(3) = 1$. 1 point requires an implicit or explicit reference to the definition of a function AND support by referencing specific function values (e.g. citing $f(-3) = f(0) = 1$); saying only 'f fails the horizontal line test' or 'f is not one-to-one' is NOT sufficient. This point cannot be earned if part (i) has any error.