

Exercise walk-through

Rational Functions and Holes ■ 1.10

eXercise

You must be able to

- identify a **hole** 空心点 where a factor cancels from numerator and denominator
- find the location (c, L) of a hole

Method — Holes

A rational function has a **hole** at $x = c$ when a factor $(x - c)$ **cancels** — the numerator's multiplicity of c is at least the denominator's.

Method — Location

The hole is at (c, L) , where $L = \lim_{x \rightarrow c} r(x)$ — evaluate the **simplified** function at c .

Method — Example

$r(x) = \frac{(x-2)(x+1)}{x-2}$ simplifies to $x+1$ (for $x \neq 2$), so there is a hole at $(2, 3)$.

Practice 2.1 ■ 1 mark

State when a rational function has a hole.

Solution 2.1

Method: Holes

Worked steps

when a factor cancels from both the numerator and the denominator **B1**.

Practice 2.2 ■ 2 marks

For $r(x) = \frac{(x-2)(x+1)}{x-2}$, state the location of the hole.

Solution 2.2

Worked steps

simplifies to $x + 1$; hole at $(2, 3)$ **M1** **A1**.

Practice 2.3 ■ 1 mark

State the difference between a hole and a vertical asymptote.

Solution 2.3

Worked steps

at a hole the function is merely undefined at one point; at an asymptote the values blow up to $\pm\infty$ **B1**.

Exam-style 3.1 ■ 1 mark

A rational function has a hole where a factor

- A** appears only in the denominator
- B** cancels from numerator and denominator
- C** is the leading term
- D** is constant

Solution 3.1

Method: Holes

- A appears only in the denominator
- B cancels from numerator and denominator** 
- C is the leading term
- D is constant

Worked steps

B.

Exam-style 3.2 ■ 1 mark

The x -value of a hole is

- A** always 0
- B** where a common factor is zero
- C** a vertical asymptote
- D** undefined for all x

Solution 3.2

- A always 0
- B where a common factor is zero
- C a vertical asymptote
- D undefined for all x



Worked steps

B.

Exam-style 3.3 ■ 1 mark

$$r(x) = \frac{(x-3)(x+4)}{x-3}.$$

- (a) Simplify $r(x)$ (for $x \neq 3$).
- (b) State the x -coordinate of the hole.
- (c) Find the y -coordinate of the hole.

Solution 3.3

Method: Example

Worked steps

(a) $x + 4$ **B1**. (b) $x = 3$ **B1**. (c) $3 + 4 = 7$ **B1**.