

2 Logarithms and inverse functions – Part 2

Name: _____ Class: _____ Date: _____

Vocabulary

Write each word three times.

English

Write it 3 times (English)

inverse function

logarithm

Recall

In Part A you met exponential growth. Now the tool that “undoes” it:

- You know that $2^3 = 8$.
- A **logarithm** answers the reverse question: what power gives 8?
- Working backwards through a function is called finding its inverse.

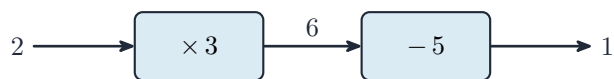
Each idea has a worked example before the Practice.

New ideas

■ 1 Inverse functions

An **inverse function** undoes what a function does: if $f(a) = b$, then $f^{-1}(b) = a$. Its graph is the reflection of f in the line $y = x$.

$f(x) = 3x - 5$: do $\times 3$, then -5



$$f^{-1}(x) = \frac{x + 5}{3}$$



reverse the order, undo each step

the inverse runs it backwards: $f(2) = 1$, so $f^{-1}(1) = 2$

■ 2 Logarithms

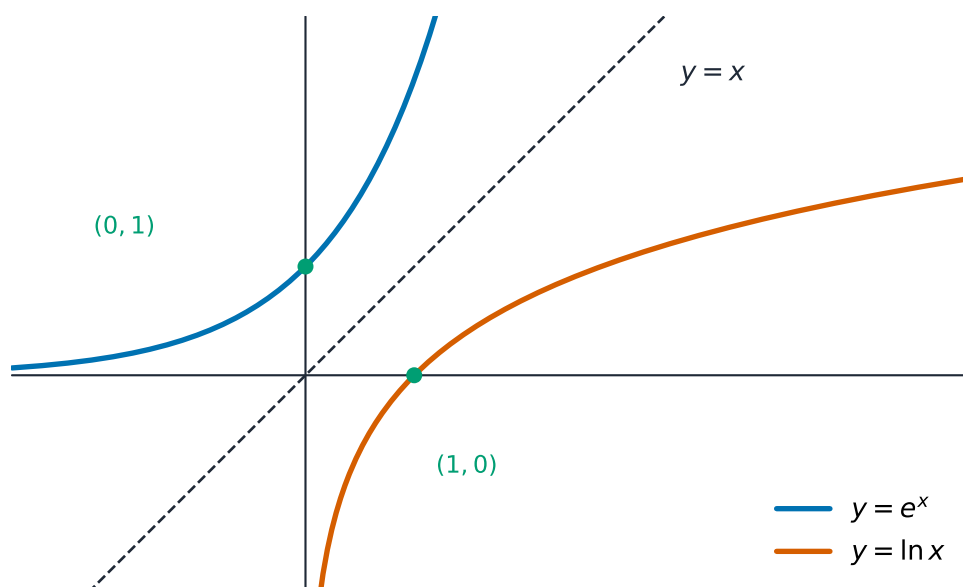
A **logarithm** answers “what exponent?”:

$$\log_b c = a \quad \text{means exactly} \quad b^a = c.$$

Worked example. $\log_2 8 = 3$, because $2^3 = 8$. And $\log_{10} 1000 = 3$, because $10^3 = 1000$.

■ 3 Logs undo exponentials

The logarithm is the **inverse of the exponential** – they cancel each other. So $\log_b(b^x) = x$ and $b^{\log_b x} = x$.



■ 4 Solving exponential equations

Because they are inverses, you can use one to undo the other.

Worked example. Solve $2 \cdot 3^x = 54$: divide by 2 to get $3^x = 27 = 3^3$, so $x = 3$.

Watch out: you can only take the logarithm of a **positive** number – $\log_b(0)$ and logs of negatives are undefined. Always check that the thing inside a log stays positive.

Practice

Try these in order. The methods are all above.

2.1 State, in words, what $\log_b c = a$ means. [1]

2.2 Find $\log_2 16$. [2]

2.3 Find $\log_{10} 100$. [1]

2.4 Solve $5^x = 125$. [2]

2.5 Solve $3 \cdot 2^x = 48$. [3]