1 Polynomials and rational functions – Part 2

Name: _____ Class: _____ Date: _____

Vocabulary

Write each word three times.

English	**Write it 3 times** (English)		
polynomial			
degree			
zero			
end behavior			
rational function			
vertical asymptote			
horizontal asymptote			

Recall

In Part A you met functions and their rates of change. Now two important families:

- **Polynomials** like $x^3 - 2x + 1$ – sums of powers of x.
- **Rational functions** – one polynomial divided by another.

Each idea has a worked example before the Practice.

New ideas

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1 Polynomials

A **polynomial** is a sum of powers of x, like $p(x) = 2x^3 - 5x + 1$. Its highest power is the **degree**, and the term with that power is the **leading term**.

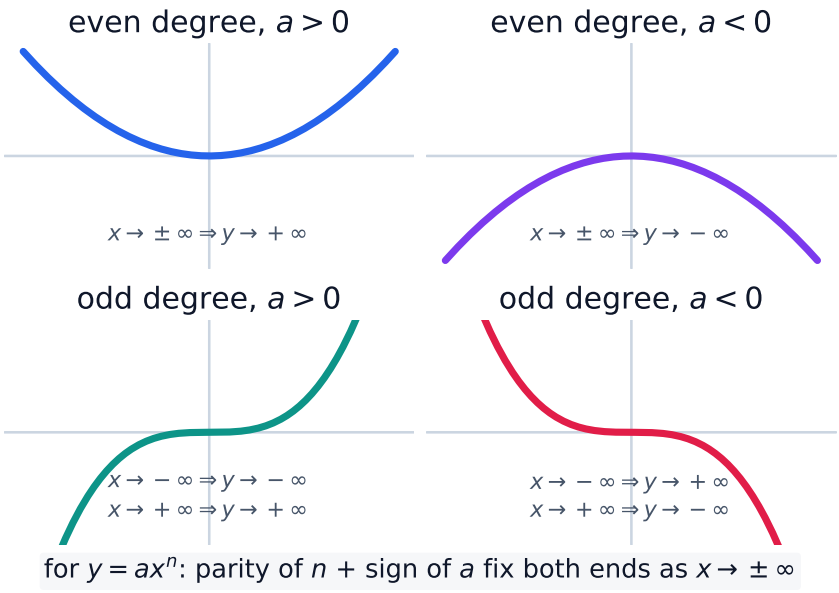
2 Zeros

A **zero** (or root) of p is an input where $p(x) = 0$. Each real zero gives an **x-intercept** where the graph crosses the x-axis.

Worked example. $p(x) = (x - 2)(x + 3)$ has zeros at $x = 2$ and $x = -3$, so its graph crosses the x-axis at those two points.

3 End behavior

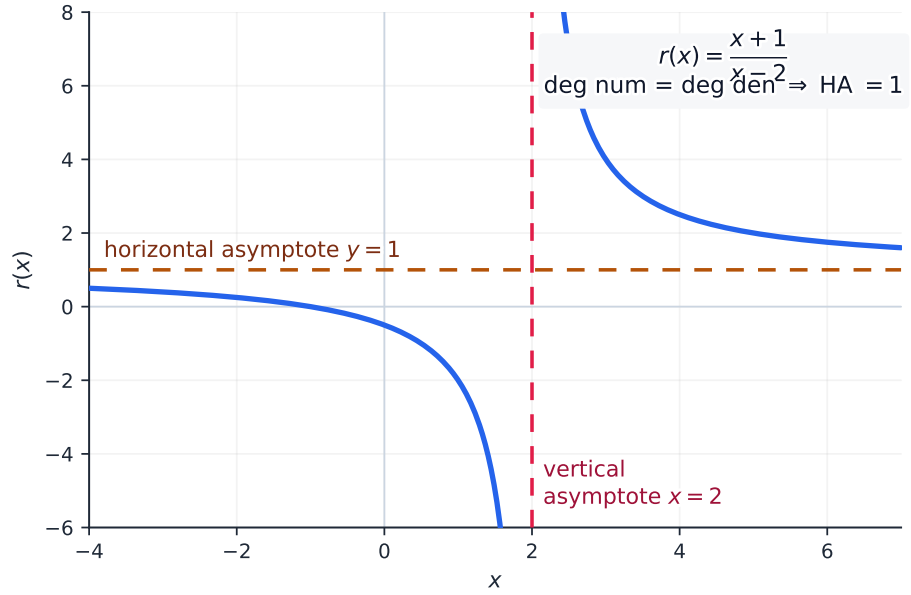
The **end behavior** – where the graph heads as $x \to \pm\infty$ – is decided entirely by the leading term.



4 Rational functions and asymptotes

A **rational function** is one polynomial over another. A **vertical asymptote** appears where the denominator is zero (and the top is not), and a **horizontal asymptote** describes where the graph levels off far out.

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Worked example. For $r(x) = \dfrac{1}{x-2}$, the denominator is zero at $x = 2$, so there is a vertical asymptote there; far out the value tends to 0, so $y = 0$ is a horizontal asymptote.

Watch out: a graph can **cross** a horizontal asymptote but never crosses a **vertical** one – the function shoots off to $\pm\infty$ as it approaches a vertical asymptote.

Practice

Try these in order. The methods are all above.

2.1 State the degree of $p(x) = 4x^3 - 2x^2 + 7$. [1]

2.2 Find the zeros of $p(x) = (x-1)(x+4)$. [2]

2.3 State where the graph of $p(x) = (x-1)(x+4)$ crosses the x-axis. [1]

2.4 For $r(x) = \dfrac{1}{x-3}$, state the vertical asymptote. [1]

2.5 For $r(x) = \dfrac{5}{x+1}$, state the horizontal asymptote as $x \to \pm\infty$. [1]

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